PUBLISHING HOUSE
OF THE ROMANIAN ACADEMY

PROCEEDINGS OF THE ROMANIAN ACADEMY, Series A,
Volume 4, Number 1 /2003, pp. 00-00

STATISTICAL FILTRATION, A NEW AND EFFICIENT METHOD FOR ULTRASONIC SIGNAL FILTRATION

Ioan RUGINA, Cristian RUGINA

Institute of Solid Mechanics , C.Mille 15, Bucharest, ROMANIA
Corresponding author: Ioan Rugina, E-mail: rugina @mecsol.ro

A new and efficient method of signal filtration in ultrasonic systems is proposed. This kind of filtration, can be called "statistical filtration", because it is based on the statistical functions of correlation theory. The method is based on the transfer of needed information about the supervised characteristic, from the signals, on their correlation functions. It gives some good results in precision improvement of different characteristic determinations in ultrasonic systems The method uses different types of correlation theory functions to improve the ratio between the signal containing the needed information about the supervised characteristic, and the noise or perturbation signal. In this way, one can obtain a substantial diminution of the errors in the determination of the needed characteristics obtained by the signal analysis, especially in ultrasonic systems.

Key words: Statistical signal filtration. Dynamic characteristic determinations Measurement precision improvement.

1. INTRODUCTION

To improve the ultrasonic system characteristic determination [2], [3], some classic signal filtering methods are usually used. These methods cannot be so easy used in all cases. If the signal represents a quasi-stationary or a non-stationary process, other methods can be used. Even if the signal represents a stationary process, some good results can be also obtained, in some cases by other methods, especially by means of so-called "statistical filtration". This method uses different types of correlation theory functions [1], to improve the ratio between the signal containing the needed information, and the noise or perturbation signal. In this way, one can obtain a substantial diminution of the errors in the determination of the needed characteristics obtained by the signal analysis [4],[5].

In the high power ultrasonic systems, the statistical filtration can be sometimes used with a good efficiency. We will show the possibility to use this method in the following two cases: the attenuation determination of a sine function having superposed some random perturbations and the determination of the phase difference between two periodical signals having the same fundamental period but including harmonics, and also some random perturbation superposed.

2. IMPROVEMENT OF ATTENUATION DETERMINATION BY MEANS OF AUTOCORRELATION FUNCTION

The possibility to use the method of statistical filtration in the case of the attenuation determination was analysed. This determination is generally performed by means of a sine signal, decreasing in time, which represents the variation of the vibration amplitude at the end of a resonator constituted by the analyzed dissipative solid material. Its logarithmic decrement, given by two consecutive maximum values of the sine signal, determines the needed attenuation value. Unfortunately, some random perturbations given by the

---

Recommended by Radu P.VOINEA
Member of the Romanian Academy

internal noise of electronic measuring system or by random vibrations of the ultrasonic system components under the control, are almost already superposed on the decreasing sine signal. Using the autocorrelation function of this signal, one can improve the ratio between the signal containing the needed information on the attenuation, and the noise or perturbation signal.

If we consider an attenuated sine signal, having superposed some random perturbations, we can describe it in the following form:

$$f(t) = e^{-\alpha t} \sin \omega t + \varepsilon(t) \tag{1}$$

where:

α - the attenuation coefficient

ω - angular frequency

$\varepsilon(t)$ - a small random perturbation

The attenuation coefficient α, can be obtained by means of two consecutive maximum values of eq.(1), at the moments t_n and t_{n+1}, for $\varepsilon(t)= 0$. We will have:

$$\alpha = \frac{\ln[f(t_n)] - \ln[f(t_{n+1})]}{t_{n+1} - t_n} \tag{2}$$

But the random perturbation $\varepsilon(t)$, affects the correct values $f(t_n)$ and $f(t_{n-1})$, giving some errors which can be reduced by means of correlation function.

The autocorrelation function $R(\tau)$ of $f(t)$, can be well approximated [1], over a large period of time T, by:

$$R(\tau) = \frac{1}{T}\int_0^T \left[e^{-\alpha t} \sin \omega t + \varepsilon(t)\right] \cdot \left[e^{-\alpha(t+\tau)} \sin \omega(t+\tau) + \varepsilon(t+\tau)\right] \mathrm{d}t \tag{3}$$

By developping the eq.3, and neglecting some terms, having very small values, one can write:

$$R(\tau) \cong e^{-\alpha\tau} \frac{1}{T}\int_0^T e^{-2\alpha t} \sin \omega t \cdot \sin \omega(t+\tau) \mathrm{d}t \tag{4}$$

After some routine calculation, one can find:

$$R(\tau) \cong \left[e^{-\alpha\tau} \sin(\omega\tau + \varphi)\right] \cdot \frac{\mathrm{I}_2}{T \cos \varphi} \tag{5}$$

with:

$$j = \mathrm{arctg} \frac{I_1}{I_2} \tag{6}$$

and:

$$I_1 = \frac{e^{-2\alpha T}}{4\alpha(\omega^2 + \alpha^2)}\left[2\omega^2 - 2\alpha \sin \omega T(\alpha \sin \omega T + \omega \cos \omega T)\right] \tag{7}$$

$$I_2 = \frac{e^{-2\alpha T}}{4(\omega^2 + \alpha^2)}\left[\omega - (\alpha \sin 2\omega T + \omega \cos 2\omega T)\right] \tag{8}$$

One can see that eq. (1) and eq. (5) represent the same exponential function $e^{-\alpha t}$, which modulate a sine function having the same period, given by the same angular frequency ω. So, their logarithmic decrement α, will be almost the same, but the factor $I_2 / T \cos \varphi$, depending also from α, can introduce some small errors.

The most important fact is that the errors, given by the random perturbations superposed on the signal, can be considerably reduced. Fig.1a shows an attenuated sine signal, obtained in our experiments, with some random perturbations, superposed, which can be expressed by eq. (1). Fig.1b shows its autocorrelation function. One can see that the random perturbations are pretty well filtrated by autocorrelation function, so its error component in the α determination, by means of eq.(2), can be substantially reduced.

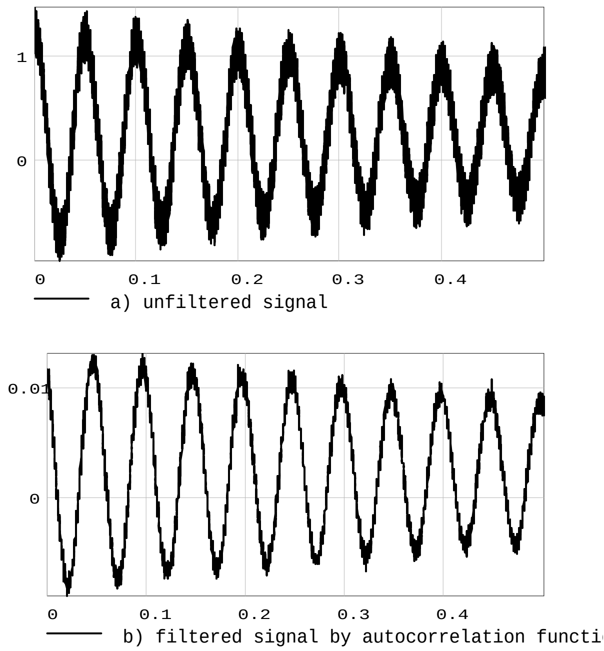

Fig.1 Attenuated sine signals.

This method can be successfully used to improve the precision in attenuation determination obtained for different components of the ultrasonic systems. In this case the logarithmic decrement given by two, or more consecutive maximum values, of the modulated sine function, having small differences between them, can be easily affected even by small random perturbation, and so the statistical filtration by means of autocorrelation function can improve considerably the precision in attenuation determination.

The results obtained for the attenuation coefficient α, given by eq.(2), in the case of some components of a high power ultrasonic transducer, constituted from aluminum, confirmed that we could obtain a precision improvement from 4% to 1.2%. The precision improvement was appreciated by the reduction of the variance, for the α value, obtained for different two pairs of the moments t_n and t_{n+1}, corresponding of two consecutive maximum values of the analyzed signal.

3. STATISTICAL FILTRATION OF SIGNALS USED IN PHASE MEASUREMENTS

The phase measurement is a very important problem in the ultrasonic systems, in connection with their power determination. In fact, because of high intensity power levels working in such a system, which can produce significant lose of power, one has to keep the ultrasonic system under control, by power measurement, to assure, in this way, its reliability.

But difficulties in power determination are given, especially, by the phase measurements. The power is generally expressed, not only by the product between two monochromatic signal amplitudes, current and tension, having the same frequency, but depends also from the phase between them. The difficulty arises when the two signals are not monochromatic. Electronic generators, working in commutation regime, generally supply the ultrasonic transducers. This fact implies that the signals are strongly distorted. Also the signals have many perturbations superposed on them. In this conditions the precision of phase measurements can be strongly reduced. The use of correlation functions can considerably improve the phase measurements in such cases.

Let us consider the two following functions:

$$x(t) = X \cdot \sin \omega t + \varepsilon_x(t) \tag{9}$$

$$y(t) = Y \cdot \sin(\omega t + \varphi) + \varepsilon_y(t) \tag{10}$$

where $\varepsilon_x(t)$ and $\varepsilon_y(t)$ are random perturbations having small values in respect with monochromatic components $X \cdot \sin \omega t$ and $Y \cdot \sin(\omega t + \varphi)$, characterized by a difference phase angle φ between them.

Their cross-correlation function $R_{yx}(\tau)$, can be approximated by:

$$R_{yx}(\tau) = \frac{1}{T}\int_0^T y(t)x(t+\tau)\mathrm{d}t = \frac{XY}{2}\cdot\frac{1}{T}\int_0^T \{\sin(\omega t+\varphi)+\varepsilon_y(t)\}\cdot\{\sin[\omega(t+\tau)]+\varepsilon_x(t)\}\mathrm{d}t \tag{11}$$

After some routine calculation, by neglecting some terms, having very small relative values, one can find:

$$R_{yx}(\tau) \cong \frac{XY}{2}\cos(\omega\tau - \varphi) \tag{12}$$

For $\tau = 0$, we will have:

$$R_{yx}(0) = \frac{XY}{2}\cos\varphi \tag{13}$$

and for $\tau = \frac{\varphi}{\omega}$, we will have:

$$R_{yx\max} = \frac{XY}{2} \tag{14}$$

So the phase difference angle φ, will be:

$$\varphi = \arccos\frac{R_{yx}(0)}{R_{yx\max}} \tag{15}$$

Hence the ratio between the values of the cross-correlation function at the two mentioned arguments gives the needed value. Because it is computed by the cross-correlation function on a very long period of time *T*, the φ-value will be much more precise, the effect of random perturbation being strongly reduced.

But not only the random components $\varepsilon_x(t)$ and $\varepsilon_y(t)$, of signals $x(t)$ and $y(t)$, can introduce some phase angle errors, in the classic determination of their phase difference φ. Also, some higher harmonics of each signals, or even a small direct current component, superposed on $x(t)$, or on $y(t)$, can introduce the same difficulty: the reduction of the precision in φ determination. Fortunately, the influence of all these components are substantially reduced in the cross-correlation function $R_{yx}(\tau)$, and so, one can improve the precision in the φ determination, by the statistical filtering method. In fact, a similar calculus of $R_{yx}(\tau)$, performed with the same signals given by eq.(9) and eq.(10), but containing besides the random components $\varepsilon_x(t)$ and $\varepsilon_y(t)$, other components, like different harmonics or some direct current component, superposed on $x(t)$, or on $y(t)$, shows that their contribution in the values of cross-correlation function $R_{yx}(\tau)$, are pretty small, and this contribution diminish with the increasing of time integration period T.

Fig.2, presents two signals, given by eq.(9) and eq.(10), respectively, having the phase difference angle φ. Both signals contain some random perturbations superposed on them, but their cross-correlation function $R_{yx}(\tau)$, presented in fig.3, is clean enough, to permit the determination, with sufficient precision, its particular values $R_{yx}(0)$, and $R_{yx\max}$, which determine the value of phase difference angle φ.

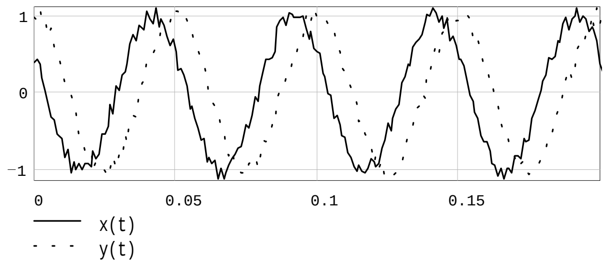

Fig.2.Sine signals having some random perturbations.

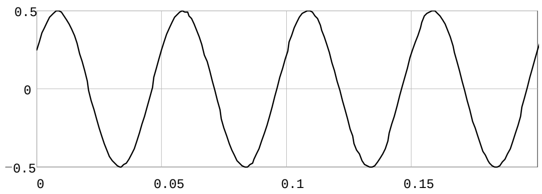

Fig.3. Cross-correlation function of two sine signals, having some random perturbations

The importance of precision improvement in phase measurement for power determination in ultrasonic systems, can be shown by analyzing the error propagation in this case.

Let:

$$P = P(U, I, \varphi) = \frac{U \cdot I \cdot \cos\varphi}{2} \tag{16}$$

be the power, given by an electronic generator into an ultrasonic system, with a voltage amplitude U, a current amplitude I, and a phase difference φ between the voltage and current signals, having the same time period.

From eq.(16), one can obtain:

$$\mathrm{d}P = \frac{\partial P}{\partial U}\mathrm{d}U + \frac{\partial P}{\partial I}\mathrm{d}I + \frac{\partial P}{\partial \varphi}\mathrm{d}\varphi \tag{17}$$

where:

$$\frac{\partial P}{\partial U} = \frac{I.\cos\varphi}{2} \quad ; \quad \frac{\partial P}{\partial I} = \frac{U.\cos\varphi}{2} \quad ; \quad \frac{\partial P}{\partial \varphi} = -\frac{U\ I\sin\varphi}{2} \tag{18}$$

Eqs.(18), can also have the following form:

$$\frac{\mathrm{d}P}{P} = \frac{U}{P}\frac{\partial P}{\partial U}\frac{\mathrm{d}U}{U} + \frac{I}{P}\frac{\partial P}{\partial I}\frac{\mathrm{d}I}{I} + \frac{\varphi}{P}\frac{\partial P}{\partial \varphi}\frac{\mathrm{d}\varphi}{\varphi} \tag{19}$$

Defining the normalized errors, ε_P, ε_U, ε_I, ε_φ, concerning the values of P, U, I and φ respectively, by:

$$\varepsilon_P = \frac{\mathrm{d}P}{P} \quad ; \ \varepsilon_U = \frac{\mathrm{d}U}{U} \quad ; \ \varepsilon_I = \frac{\mathrm{d}I}{I} \quad ; \ \varepsilon_\varphi = \frac{\mathrm{d}\varphi}{\varphi} \tag{20}$$

and taking into account that, in the error analysis one must take absolute value of all coefficients of independent variables on the right hand side (component errors always add, they never cancel each other out), one can obtain from eqs.(18),(19) and (20):

$$\varepsilon_P = \varepsilon_U + \varepsilon_I + \varphi \cdot \operatorname{tg}\varphi \cdot \varepsilon_\varphi \tag{21}$$

Eq.(21) shows the kind of phase error propagation, into the power error. One can observe that the phase error can become the major source of error in power determination, especially in ultrasonic systems, where φ can have values grater than $\pi/4$. In fact, if $\varphi > \pi/3$, the influence of ε_φ , in the value of total error, grows rapidly, and can become more than two times in respect with the ε_U and ε_I values.

So the precision improvement in the phase determination, can improve substantially the precision in power determination.

4. REMARKS AND CONCLUSIONS

The method of signal filtration, using the statistical functions of correlation theory, is based on the transfer of needed information about the supervised characteristic, from the signals, on their correlation functions. It gives some good results in precision improvement of different characteristic determinations in ultrasonic systems The method improves the ratio between the signal containing the needed information about the supervised characteristic, and the noise or perturbation signal. In this way, one can obtain a substantial diminution of the errors in the determination of the needed characteristics obtained by the signal analysis, especially in ultrasonic systems.

The statistical filtration of the signals, containing the needed information's about the attenuation, or about phase angle , by means of correlation functions can eliminate an important part of errors introduced by random perturbations of electronic measuring system or by random vibrations of the ultrasonic system components, which are under the control.

The precision improvement in this method substantially grows, if the integration period of time T, in the correlation function estimations, becomes larger, which implies more computation time. In fact, this statistical filtration method became possible, due to the computer technique, which gives also the possibility to optimize the implementation of correlation functions [7].

Even if, in some cases, the statistical filtering of the signals, can introduce itself some small errors, given by some modifications of the signal characteristics implied in the values under the control, in the process of transfer of needed information about the supervised characteristic, from the signals, on their correlation functions, however, in many cases, we can considerably reduce the total error, and so we can obtain an important improvement of the precision in the needed characteristic determination.

REFERENCES

1. BENDAT, J.S., .PIERSOL, A.G., “Engineering Applications of Correlation and Spectral Analysis”, John Wiley&Sons, 1980.
2. TRUELL,R., ELBAUM,C., CHICK,B., Ultrasonic Methods in Solid State Physics, Academic Press, New York and London, 1969.
3. PUSKAR, A., “The use of high-intensity ultrasonic”, Elsevier Co., Amsterdam, 1982.
4. RUGINA,I., CHIROIU, V., RUGINA,C.,. CHIROIU, C. “Precision improvement of ultrasonic system characteristics determination by correlation methods”, The Annual Symposium of the Institute of Solid Mechanics, SISOM ’99,Bucuresti 1999, pp. 81-86.
5. RUGINA, I., RUGINA**,** C**.,** "Dynamic behaviour analysis of the ultrasonic systems by means of statistical methods", The Annual Symposium of the Institute of Solid Mechanics SISOM‘2000, Bucharest, November 26-27, 2000, pp. 21-26
6. RUGINÃ**,**I.**,**.RUGINÃ.C., **”**Precision improvement of attenuation determination in solids by means of statistical filtration” 17-th International Congress on Acoustics, Rome,September 2-7 2001.
7. NICOLAE,M., RUGINÃ,I., VASILE,A., "Optimizing Implementation of Autocorrelation Function", The Annual Symposium of the Institute of Solid Mechanics SISOM‘2000, Bucharest, October 26-27, 2000, pp. 118-122.

Receiving date: September 2, 2002