Matrix relations for kinematics and inverse dynamics of the 3-RRR planar parallel robot are established in this paper. Three identical planar legs connecting to the moving platform are located in the same plane. Knowing the general motion of the platform, we develop first the inverse kinematics problem and determine the positions, velocities and accelerations of the robot’s elements. The inverse dynamics problem is solved using the principle of virtual work, but it has been verified the results in the framework of the Lagrange equations with multipliers. Recursive equations offer expressions and graphs for the input powers of three revolute actuators and the internal forces in the joints.

**Key words:** Kinematics, Dynamics, Planar parallel robot, Lagrange equations, Virtual work.

**LIST OF SYMBOLS**

- $a_{k,k-1}$ – orthogonal transformation matrix
- $\varphi_{k,k-1}$ – relative rotation angle of $T_k$ rigid body
- $\vec{\omega}_{k,k-1}$ – relative angular velocity of $T_k$
- $\vec{\omega}_{k,k-1}$ – skew-symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$
- $\vec{r}_{k,k-1}$ – relative position vector of the centre $A_k$ of joint
- $m_k$, $J_k$ – mass and symmetric matrix of tensor of inertia of $T_k$ about the link-frame $x_k,y_k,z_k$
- $p_{10}^a$, $p_{10}^b$, $p_{10}^c$ – powers of three fixed revolute actuators

**1. INTRODUCTION**

Compared with serial manipulators, the followings are the potential advantages of parallel robots: higher kinematical precision, lighter weight and better stiffness, greater load bearing, stable capacity and suitable position of arrangement of actuators [1]. Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2]). They are used in flight simulators and more recently for Parallel Kinematics Machines. The prototype of Delta parallel robot (Clavel [3]; Tsai and Stamper [4]; Staicu [5]) as well as the Star parallel manipulator (Hervé and Sparacino [6]) are equipped with three motors which train on the mobile platform in a three-degrees-of-freedom general translation motion. Angeles [7], Wang and Gosselin [8] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three actuators.

A mechanism is said to be a planar robot if all the moving links in the mechanism perform the planar motions. In a planar linkage, the axes of all revolute joints must be normal to the plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion. Aradyfio and Qiao [9]
examined the inverse kinematics solution for the three different 3-DOF planar parallel robots. Pennock and Kassner [10] present a kinematical study of a planar parallel robot, where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints. Merlet [11] solved the forward pose kinematics problem for a broad class of planar parallel manipulators and Yang et al. [12] concentrate on the singularity analysis of a class of 3-RRR planar parallel robots developed in its laboratory.

A recursive method is introduced in the present paper, to reduce the number of equations and computation operations by using a set of matrices for kinematics and dynamics models of the planar 3-RRR parallel robot.

2. KINEMATICS ANALYSIS

Having a closed-loop structure, the planar parallel robot 3-RRR is a special symmetrical mechanism composed of three planar kinematical chains with identical topology, all connecting the fixed base to the moving platform (Fig. 1). In the actuation schema of the parallel robot (RRR) with all revolute actuators installed on the fixed base, we consider the moving platform as the output link and the elements $A_1A_2, B_1B_2, C_1C_2$ as the input links. We attach a Cartesian frame $x_0y_0z_0(T_0)$ to the fixed base with its origin located at triangle centre $O$, the $z_0$ axis perpendicular to the base. Another mobile reference frame $x_Gy_Gz_G$ is attached to the moving platform (Fig.2).

![Fig. 1 – The planar 3-RRR parallel robot.](image1)

![Fig. 2 – Kinematical scheme of first leg $A$ of the mechanism.](image2)

Considering that the moving platform is initially located at a central configuration, we note that the relative rotation of $T_k$ body with $\phi_{k-1}$ angle must be always pointing about the direction of $z_k$ axis. One of three active legs (for example leg $A$) consists of a fixed revolute joint, a moving crank $1$ of length $l_1$, mass $m_1$ and tensor of inertia $\hat{J}_1$, which has rotation about $z_1^A$ axis with the angle $\phi_{10}^A$, the angular velocity $\omega_{10}^A = \dot{\phi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A = \ddot{\phi}_{10}^A$. A new element of the leg is a rigid rod $2$ linked at the $x_2^A y_2^A z_2^A$ frame, having a relative rotation with the angle $\phi_{21}^A$, velocity $\omega_{21}^A = \dot{\phi}_{21}^A$ and acceleration $\varepsilon_{21}^A = \ddot{\phi}_{21}^A$. It has the length $l_2$, mass $m_2$ and tensor of inertia $\hat{J}_2$. Finally, a revolute joint is introduced at the moving platform, which is schematised as an equilateral triangle congruent to the base with the edge $l = r\sqrt{3}$, mass $m_3$ and tensor of inertia $\hat{J}_3$ with respect to frame $A_3x_3^A y_3^A z_3^A$. At the central configuration, we also consider
that all legs are symmetrically extended and that the angles giving the initial pose of the mechanism have following values

\[ \alpha_a = \frac{\pi}{6}, \quad \alpha_b = \frac{5\pi}{6}, \quad \alpha_c = -\frac{\pi}{2} \]  

(2.1)

We call the matrix \( A_{i,k-1}^a \), for example, the orthogonal transformation \( 3 \times 3 \) matrix of relative rotation with the angle \( \phi_{i,k-1}^a \) of link \( T_k^A \) around \( z_k^A \) axis. Using the rotation matrix \( p_{A,k-1}^a = \text{rot}(z, \phi_{i,k-1}^a) \) and pursuing the independent legs one obtains the following transformation matrices

\[ p_{i0} = p_{i0}^a \theta_i^a, \quad p_{i1} = p_{i1}^a \theta_i^a, \quad p_{i2} = p_{i2}^a \theta_i^a, \quad (p = a, b, c; \quad i = A, B, C), \]  

(2.2)

where \( \theta_i^a, \theta_i^b, \theta_i^c \) are three appropriate constant matrices. In the inverse geometric problem, the position of the mechanism is completely given through the coordinates \( x_0^G, y_0^G \) of the mass centre \( G \) and the orientation angle \( \phi \) of the movable frame \( x_G, y_G, z_G \). The orthogonal rotation matrix of the moving platform from \( x_0, y_0, z_0 \) to \( x_G, y_G, z_G \) reference system is \( R = \text{rot}(z, \phi) \).

Further, we suppose that the position vector of \( G \) centre \( T_0 = G \) and the orientation angle \( \phi \), which are expressed by following analytical functions can describe the general absolute motion of the moving platform in its vertical plane

\[ x_0^G = x_0^G (1 - \cos \frac{\pi}{3} t), \quad y_0^G = y_0^G (1 - \cos \frac{\pi}{3} t), \quad \phi = \phi (1 - \cos \frac{\pi}{3} t). \]  

(2.3)

From the conditions \( p_{0i}^a = R, \quad p_{00} = p_{22} p_{23} p_{10}, \quad p_{10} = p_{30}(t = 0) = \theta_i^a \) (with \( i = A, B, C; \quad p = a, b, c \), concerning the orientation of the platform one, the first relations between angles of rotation are obtained

\[ \phi_i^a - \phi_i^b + \phi_i^c = \phi_{i0}^a - \phi_{i21}^b + \phi_{i32}^c = \phi_{i0}^c - \phi_{i21}^c + \phi_{i32}^c = \phi. \]  

(2.4)

Six variables \( \phi_{i0}^a, \phi_{i1}^a, \phi_{i2}^a, \phi_{i0}^b, \phi_{i1}^b, \phi_{i2}^b \) will be determined from several vector-loop equations, as follows

\[ \vec{r}_i^{a} = \sum_{k=1}^{3} a_{i0}^k \vec{T}_{i1}^{k} + a_{i0}^3 \vec{T}_{i1}^{3} = \vec{r}_{i0}^a + \sum_{k=1}^{3} b_{i0}^k \vec{T}_{i3}^{k} + b_{i0}^3 \vec{T}_{i3}^{3} = \vec{r}_{i0}^b + \sum_{k=1}^{3} c_{i0}^k \vec{T}_{i3}^{k} + c_{i0}^3 \vec{T}_{i3}^{3} = \vec{r}_{i0}^c, \]  

(2.5)

where:

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}, \quad
\begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(2.6)

\[ \vec{r}_{i0}^a = 0.5r[\sqrt{3} - 1 \quad 0]^T, \quad \vec{r}_{i0}^b = r[0 \quad 0 \quad 1]^T, \quad \vec{r}_{i0}^c = 0.5r[-\sqrt{3} - 1 \quad 0]^T, \quad \vec{r}_{i0}^g = \vec{r}_i^g, \quad \vec{r}_i^g = -\vec{r}_i^g. \]

In the inverse kinematics modelling we compute first the linear and angular velocities of each leg in terms of the angular velocity \( \vec{\omega}_0^g = \vec{\phi} \vec{u}_3 \) and the centre’s velocity \( \vec{v}_0^g = \vec{r}_0^g \) of the moving platform. The kinematics of the elements of the leg \( A \) are characterized by the velocities of the centres of joints \( A_k \) and the absolute angular velocities:

\[
\vec{v}_{i0}^a = \vec{0}, \quad \vec{v}_{i20}^a = \vec{\phi}_{i20}^a \vec{a}_{i20} \vec{u}_2^a \vec{r}_{i2}^a, \quad \vec{v}_{i32}^a = \vec{a}_{i32} \vec{v}_{i20}^a + (\vec{\phi}_{i21}^a - \vec{\phi}_{i10}^a) \vec{a}_{i32} \vec{u}_2^a \vec{r}_{i2}^a.
\]

(2.7)

Equations of geometrical constraints (2.4) and (2.5) can be derive with respect to time to obtain the following matrix conditions of connectivity [13]

...
\[
\omega_0^j \ddot{u}_j + a_{10}^j \ddot{u}_j (\ddot{r}_j^T + a_{21}^j \dot{r}_j + a_{22}^j \dot{r}_j^T + a_{23}^j \dot{r}_j^T G^G) + \omega_2^j \ddot{u}_j + a_{10}^j \ddot{u}_j (\ddot{r}_j^T + a_{21}^j \dot{r}_j + a_{22}^j \dot{r}_j^T G^G) + \omega_3^j \ddot{u}_j + a_{10}^j \ddot{u}_j (\ddot{r}_j^T + a_{21}^j \dot{r}_j + a_{22}^j \dot{r}_j^T G^G) = \ddot{u}_j^G (j = 1, 2),
\]

(2.8)

From these equations, we obtain the relative velocities \( \omega_0^j, \omega_1^j, \omega_2^j \) as functions of basic velocities \( \omega_0^G, \omega_1^G, \phi^G \).

Now, let us assume that the robot has successively \textit{virtual motions} determined by three sets of angular velocities \( \omega_{00}^j = 1, \omega_{01}^j = 0, \omega_{02}^j = 0 \), \( \omega_{10}^j = 0, \omega_{11}^j = 1, \omega_{12}^j = 0 \) and \( \omega_{20}^j = 0, \omega_{21}^j = 1, \omega_{22}^j = 1 \). All characteristic virtual velocities are expressed as functions of the pose of the mechanism by the general kinematical equations (2.8). As for the accelerations \( e_{10}^d, e_{21}^d, e_{32}^d \) of the robot, new conditions of connectivity are obtained:

\[
e_{10}^d \ddot{u}_1^d a_{10}^d \ddot{u}_1^d (\ddot{r}_1^d + a_{21}^d \dot{r}_1^d + a_{22}^d \dot{r}_1^d G^G) + e_{21}^d \ddot{u}_2^d a_{10}^d \ddot{u}_2^d (\ddot{r}_2^d + a_{21}^d \dot{r}_2^d + a_{22}^d \dot{r}_2^d G^G) +
\]

\[
+ \epsilon_{32}^d \ddot{u}_3^d a_{10}^d \ddot{u}_3^d (\ddot{r}_3^d + a_{21}^d \dot{r}_3^d + a_{22}^d \dot{r}_3^d G^G) - \omega_0^d \omega_1^d \ddot{u}_1^d a_{10}^d \ddot{u}_1^d (\ddot{r}_1^d + a_{21}^d \dot{r}_1^d + a_{22}^d \dot{r}_1^d G^G) -
\]

\[
- \omega_2^d \omega_1^d \ddot{u}_2^d a_{10}^d \ddot{u}_2^d (\ddot{r}_2^d + a_{21}^d \dot{r}_2^d + a_{22}^d \dot{r}_2^d G^G) - \omega_3^d \omega_1^d \ddot{u}_3^d a_{10}^d \ddot{u}_3^d (\ddot{r}_3^d + a_{21}^d \dot{r}_3^d + a_{22}^d \dot{r}_3^d G^G) -
\]

(2.9)

The derivatives of the relations (2.7) give the accelerations \( \ddot{v}_1^d, \ddot{v}_2^d \).

3. DYNAMICS EQUATIONS

Three different methods could lead to the same results concerning the input torques. The first one is the Newton-Euler approach, which consists to apply the free-body diagram procedure for each body [14]. The second method is based on the Lagrange formalism, which introduces scalar multipliers for each closure equation and the third method for the dynamics analysis is based on the principle of virtual work [15].

3.1. Principle of virtual work

Knowing the kinematics state of each link as well as the external forces acting on the robot, one applies first the principle of virtual work for the inverse dynamic problem in order to establish some matrix relations. Three electric motors that generate the moments \( m_1^d, m_2^d, m_3^d \) oriented about fixed axes, simultaneously control the motion of the moving platform. The parallel robot can artificially be transformed in a set of three open chains by introducing the corresponding constraint conditions.

The wrench of two vectors \( \vec{F}_k^d = 9.81 m_1^d a_{10}^1 \ddot{u}_j, \vec{M}_k^d = 9.81 m_1^d \dot{r}_k^C a_{10}^1 \ddot{u}_j \) evaluates the action of the weight \( m_1^G \) and eventually of other external and internal forces applied to the same element \( T_k^d \) of the mechanism.

We compute also the force of inertia \( \vec{F}_{10}^d \) and the resulting moment of inertia forces \( \vec{M}_{10}^d \) of an arbitrary rigid body \( T_k^d \) of mass \( m_k^d \) and tensor of inertia \( \vec{J}_k^d \) with respect to the centre of its first joint:

\[
\vec{F}_{10}^d = -m_k^d \ddot{r}_1^d + (\ddot{\omega}_1^d \dot{\omega}_1^d + \ddot{\omega}_1^d \dot{\omega}_1^d) \dot{\omega}_1^d \}
\]

(3.1)

By intermediate of the conditions of connectivity (2.8), the absolute virtual velocities \( \ddot{v}_1^d, \ddot{\omega}_1^d \) are related to a set of independent \textit{relative virtual velocities}. The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Total virtual work contributed by the first input torque \( m_1^d, \) for example, inertia
forces and moments of inertia forces and by the wrench of known external or internal forces can be written in a compact form. Applying the fundamental equations of the parallel robots dynamics \[16\], a compact matrix relation results for the first torque
\[
m_{10}^A = \ddot{u}_3^T (\dot{M}_1^A + \omega_{21u}^A \dot{M}_2^A + \omega_{32u}^A \dot{M}_3^A + \omega_{21v}^B \dot{M}_2^B + \omega_{32v}^B \dot{M}_3^B).
\] (3.2)

Two recursive relations generate the vectors
\[
\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1}^T \vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1}^T \vec{M}_{k+1}^A + \vec{r}_{k+1}^A \dot{a}_{k+1}^T \vec{F}_{k+1}^A
\]
\[
\vec{F}_{k0}^A = - \vec{J}_{k0}^{adi} - \vec{J}_k^A, \quad \vec{M}_{k0}^A = - \vec{m}_{k0}^{adi} - \vec{m}_k^A \quad (k = 1, 2, 3).
\] (3.3)

### 3.2. Lagrange equations

A solution of the dynamics problem of the 3-RRR planar parallel robot can be developed based on the Lagrange equations with constraints. The generalized coordinates are represented by 12 independent parameters:
\[
q_1 = x_0^g, \quad q_2 = y_0^g, \quad q_3 = z_0^g, \quad q_s = \phi_{10}^g, \quad q_6 = \phi_{32}^g
\]
\[
q_7 = \phi_{10}^b, \quad q_8 = \phi_{21}^b, \quad q_9 = \phi_{20}^b, \quad q_{10} = \phi_{31}^b, \quad q_{11} = \phi_{32}^b, \quad q_{12} = \phi_{32}^b.
\] (3.4)

The Lagrange equations with nine multipliers \(\lambda_1, \lambda_2, \ldots, \lambda_9\) will be expressed by 12 differential relations \[17\]
\[
\frac{d}{dr} \{ \frac{\partial L}{\partial q_k} \} - \frac{\partial L}{\partial \dot{q}_k} = Q_k + \sum_{s=1}^9 \lambda_s c_{sk} \quad (k = 1, 2, \ldots, 12),
\] (3.5)

which contain following 12 generalized forces \(Q_1 = 0, Q_2 = 0, Q_3 = 0, Q_4 = m_0^i, Q_5 = 0, Q_6 = 0, Q_7 = m_0^b, Q_8 = 0, Q_9 = 0, Q_{10} = m_0^b, Q_{11} = 0, Q_{12} = 0\).

A number of nine kinematical conditions of constraint are given from the relations \(2.8\):
\[
\sum_{k=1}^{12} c_{sk} \dot{q}_k = 0 \quad (s = 1, 2, \ldots, 9).
\] (3.6)

The general Lagrange function \(L = L_p + \sum_{s=1}^2 (L_v^A + L_v^B + L_C^C)\) is expressed as analytical function of the generalized coordinates and their first derivatives with respect to time
\[
L_p = \frac{1}{2} m_p \ddot{v}_p^T \ddot{v}_p - m_p g z_0^g, \quad L_1 = \frac{1}{2} \ddot{\omega}_{10}^g \dot{\omega}_{10} - m_0^i g u_2^T p_{10}^T p_{10}^T C, \quad L_2 = \frac{1}{2} m_2 \ddot{v}_2^T \dot{v}_2 + \frac{1}{2} \ddot{\omega}_{20}^g \dot{\omega}_{20} + m_2 \ddot{v}_2^T \dot{v}_2 C - m_2 g u_2^T p_{10}^T (\dot{r}_{21}^i + p_{20}^T r_{21}^i).
\] (3.7)

The first derivatives of orthogonal matrices \(p_{1,k-1}\) are computed as follows:
\[
\ddot{p}_{1,k-1} = \ddot{\phi}_{1,k-1}^i \ddot{u}_3^T p_{1,k-1}, \quad \dot{p}_{1,k-1} = \dot{\phi}_{1,k-1}^i p_{1,k-1}^T \ddot{u}_3, \quad \frac{\partial p_{1,k-1}}{\partial q_{1,k-1}} = \ddot{u}_3^T p_{1,k-1}, \quad \frac{\partial p_{1,k-1}}{\partial q_{2,k-1}} = p_{1,k-1}^T \ddot{u}_3.
\] (3.8)

A long calculus about the partial derivatives and the expressions \(\frac{d}{dr} \{ \frac{\partial L}{\partial q_s} \}\) leads to an algebraic system of 12 relations. In the direct or inverse dynamics problem, after elimination of the nine multipliers, finally we obtain same expressions \(3.2\) for the three input torques required by the three revolute actuators.
4. CALCULUS OF INTERNAL JOINT FORCES

4.1. Principle of virtual work

Above compact relations (3.2) and (3.3) can be also applied to calculate any joint force or joint torque by cutting successively each joint and considering several appropriate virtual displacements.

For the calculus of two external forces acting in joint $A_1$, for example, we consider two different successive virtual displacements as follows

\[
\begin{align*}
\vec{v}_{10u}^{dv} &= a_{10} \vec{u}_1, \quad \omega_{10u}^{dv} = 0, \quad \omega_{10u}^{b} = 0, \quad \omega_{10u}^{c} = 0, \quad \vec{v}_{10u}^{dv} = a_{10} \vec{u}_2, \quad \omega_{10u}^{dv} = 0, \quad \omega_{10u}^{b} = 0, \quad \omega_{10u}^{c} = 0. \quad (4.1)
\end{align*}
\]

Using the matrix conditions of connectivity (2.8), we determine two new sets of independent relative virtual velocities. Finally, the recursive relations (3.2) give the two joint forces:

\[
\begin{align*}
\begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix}
&= \begin{bmatrix}
\mathbf{A}_1 \\
\mathbf{A}_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix} + \mathbf{G} \begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix}, \\
\begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix}
&= \begin{bmatrix}
\mathbf{A}_3 \\
\mathbf{A}_4
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix} + \mathbf{G} \begin{bmatrix}
\mathbf{F}_{10}^A \\
\mathbf{M}_{10}^A
\end{bmatrix}.
\end{align*}
\]

Analogously, the joint force $\mathbf{F}_{21}^A$ in $A_2$ is quickly determined if we suppose others two virtual displacement in the robot’s kinematics, starting from the basic virtual velocities

\[
\begin{align*}
\vec{v}_{21u}^{dv} &= a_{21} \vec{u}_1 \quad \text{or} \quad \vec{v}_{21u}^{dv} = a_{21} \vec{u}_2. \quad (4.3)
\end{align*}
\]

4.2. Lagrange equations

For the planar mechanical system represented by the set of 12 variables $(q_1, q_2, ..., q_{12})$, the Lagrange equations are completed with following differential relation [18]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = f_{10y}^A + \sum_{i=1}^{12} \lambda_i C_{ik} (k = 1, 2, ..., 13), \quad (4.4)
\]

where the vertical external force $f_{10y}^A$ in the joint $A_1$, for example, as new generalized force can be found if a new fictitious mobility in accord with the joint is considered.

Supposing $\vec{r}_{10u} = \vec{r}_{10} + w \vec{u}_2, \vec{v}_{10u} = w a_{10} \vec{u}_2, \vec{v}_{10w} = w a_{10} \vec{u}_2$, we determine a new expression for the Lagrange function $L_A$ and we replace it in the formula (4.4). Considering again the mechanism, the joint force is obtained in following definitive form

\[
\begin{align*}
\mathbf{F}_{10y}^A &= \left[ \frac{d}{dt} \left( \frac{\partial L_A}{\partial \dot{q}_k} \right) \right]_{q_k = 0} - (m_1^i + m_2^i) g - \lambda_13. \quad (4.5)
\end{align*}
\]

5. EXAMPLE

As application, let us consider a 3-RRR planar robot which has the following characteristics:

\[
\begin{align*}
\begin{bmatrix}
\mathbf{x}_0^* \\
\mathbf{y}_0^* \\
\phi_0^*
\end{bmatrix} &= \begin{bmatrix}
-0.025m \\
0.025m \\
\pi/12
\end{bmatrix}, \quad \mathbf{r} = 0.3m, \quad l = r \sqrt{3}, \quad l_1 = l_2 = 0.3m, \quad \Delta t = 3s, \quad m_1 = 3kg, \quad m_2 = 1.5kg, \quad m_3 = 5kg.
\end{align*}
\]

Assuming that there is no external force and moment acting on the moving platform, the numerical study is based on the computation of the power required by each actuator $P_{10}^A, P_{10}^B, P_{10}^C$ during the platform’s evolution.
Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the planar parallel robot. To illustrate the algorithm, it is assumed that for a period of three seconds the platform starts at rest from a central configuration and rotates or moves along rectilinear directions.

Following examples are solved to illustrate the simulation. If the platform’s centre $G$ moves along a rectilinear planar trajectory without rotation of platform, the powers required by the actuators $A$, $B$, $C$, are calculated by the program and plotted versus time (Fig. 3). Concerning the force $f_{1i}$ in the external joint $A$, for example, this is given as follows (Fig. 4). For the second example we consider the rotation motion of the moving platform about $z_0$ axis with variable angular acceleration while all other positional parameters are held equal to zero. The powers of the actuators (Fig. 5) and the internal joint force $f_{1i}$ in $A_i$ (Fig. 6), for example, are also determined by the program and plotted versus time.

6. CONCLUSIONS

In the inverse kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of the planar parallel robot have been established in the present paper.

The dynamics model takes into consideration the masses and forces of inertia introduced by all component elements of the parallel mechanism. Based on the principle of virtual work or the Lagrange equations, the approach establishes also a direct determination of the time-history evolution for all forces in external and internal joints.
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Received June 1, 2010