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BRIGHT AND DARK SOLITON SOLUTIONS OF THE GENERALIZED ZAKHAROV-KUZNETSOV-BENJAMIN-BONA-MAHONY NONLINEAR EVOLUTION EQUATION

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In this paper, we obtain the 1-soliton solutions of the generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony (GZK-BBM) equation. By using a solitary wave ansatz in the form of sech^p function and another wave ansatz in the form of $\tanh^p$ function we obtain bright and dark soliton solutions for this equation. The physical parameters in the soliton solutions: amplitude, inverse width, and velocity are obtained as functions of the dependent model coefficients.

Key words: exact solution, bright and dark soliton, generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation.

1. INTRODUCTION

In recent years, much effort has been spent on the construction of exact solutions to nonlinear evolution equations (NLEEs) [1–46]. The NLEEs are frequently used to describe many problems of plasma physics, diffusion process, geochemistry, protein chemistry, chemically reactive materials, mathematical biology, ecology (models of population growth), physics the heat flow, solid state physics, biology, meteorology, electricity, chemical reactions, and optical fibers. It is well known that wave phenomena of optical fibers and nonlinear dispersive media are modeled by dark shaped $\tanh^p$ solutions or by bright shaped sech^p solutions.

Getting inspiration from many applications of Benjamin-Bona-Mahony (BBM), Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZKBBM), Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KPBBM) and Korteweg-de Vries (KdV) equations in real life problems [1], several authors reported exact (closed-form) solitary solutions of these nonlinear evolution equations.

The term soliton was coined by Zabusky and Kruskal [2] after their findings that waves like particles retained their shapes and velocities after interactions [3]. The first published observation [4] of a solitary wave, i.e. a single and localized wave was made by a naval architect John Scott Russell in 1834. Russell explored his experiments in his report to the British Association for the Advancement of Science [4]. Khan *et al.* [5] found solitary wave solutions to the GZK-BBM equation. Recently, Doha *et al*. [6] developed a numerical solution for two coupled nonlinear Klein-Gordon (KG) partial differential equations.

There are several kinds of solitary solutions that are obtained in the literature. They are bright solitons, shock waves, cnoidal waves, peakons, cuspons, stumpons, and many others. The problem of finding exact solutions for nonlinear models is of great importance, both from mathematical and physical point of view. In the past decades, many methods were developed for finding exact solutions of NLEEs as the tanh-sech method [7, 8], extended tanh method [9, 10], sine-cosine method [11, 12], homogeneous balance method [13, 14], first integral method [15, 16], Jacobi elliptic function method [17, 18], (G'/G)-expansion method [19–21], F-expansion method [22, 23], and variational iteration method (VIM) [24].

A lot of experiments have been performed using solitons in optical fibers applications. The theory of optical solitons has made spectacular progress in the past few decades. There have been many advances

made in the areas of nonlinear fiber optics and nonlinear photonics [25–37]. Dark optical solitons are more stable in presence of noise and spread more slowly in presence of loss, in the optical communication systems, as compared to bright optical solitons [38–40].

The paper is organized as follows: in Section 2, we obtain the bright and dark soliton solutions of nonlinear generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony (GZK-BBM) equation and establish the families of soliton solutions. In Section 3, we briefly make a summary to the results that we have obtained.

2. THE GZK-BBM EQUATION

We consider a generalized form of the GZK-BBM equation [41]

$$u_t + u_x + \alpha(u^n)_x + \beta(u_{xt} + u_{yy})_x = 0 \ , \quad n > 1, \tag{1}$$

which is a typical nonlinear evolution equation, where α and β are real constants. Bekir [41] obtained the periodic solutions for Eq. (1) and their analytic expressions with the tanh-coth method. Next, the bright and dark soliton solution of this equation will be obtained.

2.1. The bright (non topological) soliton solution

To obtain the soliton solution of equation (1), the solitary wave ansatz admits the use of the assumption,

$$u(x, \ y, \ t) = \lambda \operatorname{sech}^p \tau, \tag{2}$$

where

$$\tau = ax + by - vt. \tag{3}$$

Here the condition

$$p > 0 \tag{4}$$

is imposed for bright soliton solutions and λ, a, b, and v are constant coefficients.

The parameter λ is the amplitude of the soliton, while a and b represent the inverse width in the x- and y-directions, respectively, and v is the velocity of the soliton. The exponent p is unknown at this point and will be determined later. From the ansatz (2) and (3) we obtain:

$$\begin{aligned} &p\lambda v \tanh\tau \operatorname{sech}^p\tau - p\lambda a \tanh\tau \operatorname{sech}^p\tau - \alpha np\lambda^n a \tanh\tau \operatorname{sech}^{np}\tau + \beta\lambda p^3 a^2 v \tanh\tau \operatorname{sech}^p\tau - \\ &- \beta p(1+p)(p+2)\lambda a^2 v \tanh\tau \operatorname{sech}^{p+2}\tau - \beta\lambda p^3 ab^2 \tanh\tau \operatorname{sech}^p\tau + \\ &+ \beta p(1+p)(p+2)\lambda ab^2 \tanh\tau \operatorname{sech}^{p+2}\tau = 0 \end{aligned} \tag{5}$$

Equating the exponents of $\tanh\tau \operatorname{sech}^{np}\tau$ and $\tanh\tau \operatorname{sech}^{p+2}\tau$ terms in Eq. (5), one obtains

$$pn = p + 2, \tag{6}$$

which implies

$$p = 2/(n-1). \tag{7}$$

By setting the corresponding coefficients of $\tanh\tau \operatorname{sech}^{np}\tau$ and $\tanh\tau \operatorname{sech}^{p+2}\tau$ terms to zero one gets

$$-\alpha np\lambda^n a \ -\beta p(1+p)(p+2) \ \lambda a^2 v \ +\beta p(1+p)(p+2) \ \lambda ab^2 \ = 0. \tag{8}$$

We next obtain

$$v = \frac{-\alpha n \lambda^{n-1} + \beta b^2 (1+p)(p+2)}{a\beta(1+p)(p+2)}. \tag{9}$$

Setting the coefficients of $\tanh\tau \operatorname{sech}^p \tau$ terms in Eq. (5) to zero we obtain

$$p\lambda v \quad -p\lambda a + \beta \quad \lambda p^3 a^2 v \quad -\beta\lambda p^3 a b^2 \quad = 0. \tag{10}$$

Solving Eq. (10) by using Eq. (9) we obtain

$$\lambda = \left(\frac{\beta(3pb^2 + 2b^2 + p^2 b^2 - p^2 a^2 - 3pa^2 - 2a^2)}{\alpha n(1 + p^2 a^2 \beta)} \right)^{\frac{1}{n-1}}. \tag{11}$$

Therefore the 1-soliton solution of the GZK-BBM equation is given by

$$u(x, \ y, \ t) = \lambda \operatorname{sech}^{\frac{2}{n-1}} \ (ax + by - vt), \tag{12}$$

where the velocity of the soliton is given by Eq. (9) and the amplitude-width relation of the soliton is given by Eq. (11).

Case I: $p = 2$. This yields

$$n = 2 \tag{13}$$

so that the GZK-BBM equation given by (1) modifies to

$$u_t + u_x + \alpha(u^2)_x + \beta(u_{xt} + u_{yy})_x = 0 \tag{14}$$

Further substitution of $p = 2$ and $n = 2$ into (9) and (11) gives, respectively,

$$v = \frac{-\alpha\lambda + 6\beta b^2}{6a\beta}, \tag{15}$$

$$\lambda = \frac{6\beta(b^2 - a^2)}{\alpha(1 + 4a^2\beta)}. \tag{16}$$

Thus, in this case, the bright soliton solution is given by

$$u(x, \ y, \ t) = \left(\frac{6\beta(b^2 - a^2)}{\alpha(1 + 4a^2\beta)} \right) \operatorname{sech}^2 \ (ax + by - \frac{-\alpha\lambda + 6\beta b^2}{6a\beta} t). \tag{17}$$

The above bright soliton solution exists provided that

$$1 + 4a^2\beta \neq 0. \tag{18}$$

Case II: $p = 1$. This yields

$$n = 3 \tag{19}$$

so that the GZK-BBM equation given by (1) modifies to

$$u_t + u_x + \alpha(u^3)_x + \beta(u_{xt} + u_{yy})_x = 0. \tag{20}$$

Khan *et al.* [5] have successfully implemented the adequate method to find the exact (closed-form) solitary wave solutions for the GZK-BBM equation of this type. Further substitution of $p = 1$ and $n = 3$ into Eq. (9) and Eq. (11) gives, respectively,

$$v = \frac{-\alpha\lambda^2 + 2\beta b^2}{2a\beta}, \quad \beta \neq 0, \tag{21}$$

$$\lambda = \pm\sqrt{\frac{2\beta(b^2 - a^2)}{\alpha(1 + a^2\beta)}}, \quad \alpha \neq 0. \tag{22}$$

Thus, in this case, the bright soliton solution is given by

$$u(x,\ y,\ t) = \pm\sqrt{\frac{2\beta(b^2 - a^2)}{\alpha(1 + a^2\beta)}}\operatorname{sech}\left\{ax + by - \frac{-\alpha\lambda^2 + 2\beta b^2}{2a\beta}t\right\}. \tag{23}$$

Finally, it is necessary to have

$$n \neq 1 \tag{24}$$

as seen from Eqs. (12), (16), and (17). Again from Eqs. (4) and (12), it can be concluded that it is necessary to have

$$n > 1 \tag{25}$$

for the soliton to exist. On the other hand we see that the solitons will exist when

$$\alpha\beta(b^2 - a^2)(1 + a^2\beta) > 0. \tag{26}$$

2.2. The dark (topological) soliton solution

In this section, we are interested in finding the dark solitary wave solution, as defined in Ref. [26] for the considered the GZK-BBM equation (1). In order to construct dark soliton solutions for Eq. (1), we use an ansatz solution of the form [42]

$$u(x,\ y,\ t) = \lambda\tanh^p\tau, \tag{27}$$

where

$$\tau = ax + by - vt \tag{28}$$

and the condition

$$p > 0 \tag{29}$$

is necessary for the existence of dark soliton solutions. Here in Eq. (27) and Eq. (28), λ, a, and b are unknown free parameters and v is the velocity of the soliton that will be determined. The exponent p is also unknown. From Eq. (27) and Eq. (28) we have:

$$\begin{aligned} &p\lambda v \ \{\tanh^{p+1}\tau - \tanh^{p-1}\tau\} + p\lambda a\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\} + \alpha np\lambda^n a \ \{\tanh^{pn-1}\tau - \tanh^{pn+1}\tau\} - \\ &-\beta p \ \lambda a^2 v \ \left\{\begin{matrix}(p-1)(p-2)\tanh^{p-3}\tau - [2p^2 + (p-1)(p-2)]\tanh^{p-1}\tau + \\ +[2p^2 + (p+1)(p+2)]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau\end{matrix}\right\} + \\ &+\beta p\lambda ab^2 \ \left\{\begin{matrix}(p-1)(p-2)\tanh^{p-3}\tau - [2p^2 + (p-1)(p-2)]\tanh^{p-1}\tau + \\ +[2p^2 + (p+1)(p+2)]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau\end{matrix}\right\} = 0. \end{aligned} \tag{30}$$

Equating the highest exponents of $\tanh^{np+1}\tau$ and $\tanh^{p+3}\tau$ terms in Eq. (30), one gets

$$pn + 1 = p + 3, \tag{31}$$

which yields the following analytical condition:

$$p = 2/(n-1). \tag{32}$$

The same value of p is yielded on equating the exponents $pn - 1$ and $p + 1$. By setting the corresponding coefficients of $\tanh^{np+1}\tau$ and $\tanh^{p+3}\tau$ terms to zero one gets

$$-\alpha np\lambda^n a \quad +\beta p(1+p)(p+2) \quad \lambda a^2 v \quad -\beta p(1+p)(p+2) \quad \lambda ab^2 \quad = 0. \tag{33}$$

The condition (33) gives

$$v = \frac{(1+p)(p+2)\beta b^2 + \alpha n\lambda^{n-1}}{a\beta(p+1)(p+2)} \quad . \tag{34}$$

Setting the coefficients of $\tanh^{np+1}\tau$ and $\tanh^{p+3}\tau$ terms in Eq. (30) to zero we have

$$p\lambda v - p\lambda a + \alpha\lambda^n npa - [2p^2 + (p+1)(p+2)]\beta\, p\lambda v a^2 + [2p^2 + (p+1)(p+2)]\beta p\lambda a b^2 = 0. \tag{35}$$

Solving Eq. (35) by using Eq. (34) we obtain

$$\lambda = \left(-\frac{\beta(2a^2 + p^2a^2 + 3pa^2 - 3pb^2 - 2b^2 - p^2b^2)}{\alpha n(2p^2a^2\beta - 1)} \right)^{\frac{1}{n-1}}. \tag{36}$$

Finally, the dark (topological) soliton solution of the GZK-BBM equation is given by

$$u(x,\ y,\ t) = \lambda \quad \tanh^{\frac{2}{n-1}} \quad (ax + by - vt), \tag{37}$$

where the velocity of the soliton is given by (34) and connection between the free parameters of the soliton is seen in Eq. (36).

Case I: $p = 2$. This yields

$$n = 2 \tag{38}$$

so that the GZK-BBM equation given by Eq. (1) modifies to

$$u_t + u_x + \alpha(u^2)_x + \beta(u_{xt} + u_{yy})_x = 0. \tag{39}$$

Further substitution of $p = 2$ and $n = 2$ into Eq. (34) and Eq. (36) gives, respectively,

$$v = \frac{\alpha\lambda + 6\beta b^2}{6a\beta}, \quad \beta \neq 0,\ a \neq 0, \tag{40}$$

$$\lambda = \frac{6\beta(b^2 - a^2)}{\alpha(8a^2\beta - 1)}, \quad \alpha \neq 0. \tag{41}$$

Thus, in this case, the soliton solution is given by

$$u(x,\ y,\ t) = \left(\frac{6\beta(b^2 - a^2)}{\alpha(8a^2\beta - 1)} \right) \tanh^2\left(ax + by - \frac{\alpha\lambda + 6\beta b^2}{6a\beta} t\right). \tag{42}$$

The above soliton solution exists provided that

$$8a^2\beta - 1 \neq 0. \tag{43}$$

Case II: $p = 1$. This yields

$$n = 3 \tag{44}$$

so that the GZK-BBM equation given by Eq. (1) modifies to

$$u_t + u_x + \alpha(u^3)_x + \beta(u_{xt} + u_{yy})_x = 0. \tag{45}$$

Similarly taking $p = 1$ and $n = 3$ into Eq. (34) and Eq. (36) gives, respectively,

$$v = \frac{\alpha\lambda^2 + 2\beta b^2}{2a\beta}, \quad \beta \neq 0, \tag{46}$$

$$\lambda = \pm\sqrt{\frac{2\beta(b^2 - a^2)}{\alpha(2a^2\beta - 1)}}, \quad \alpha \neq 0. \tag{47}$$

Thus, in this case, the dark soliton solution is given by

$$u(x,\ y,\ t) = \pm\sqrt{\frac{2\beta(b^2 - a^2)}{\alpha(2a^2\beta - 1)}}\ \tanh\left\{ax + by - \frac{\alpha\lambda^2 + 2\beta b^2}{2a\beta}t\right\}. \tag{48}$$

Finally, it is necessary to have

$$n \neq 1 \tag{49}$$

as can be seen from Eqs. (32), (36), and (37). Again from Eq. (29) and Eq. (32), it can be concluded that it is necessary to have

$$n > 1 \tag{50}$$

for the existence of soliton solutions.

On the other hand from Eq. (47) we see that the solitons will exist for

$$\alpha\beta(b^2 - a^2)(2a^2\beta - 1) > 0. \tag{51}$$

3. CONCLUSIONS

In this paper we have obtained both topological and non-topological soliton solutions of the generalized Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation. The solitary wave ansatz was used to carry out the integration of this nonlinear evolution equation. Exact 1-soliton solutions were obtained. They are topological solitons that are also known as shock waves, and non-topological (bright) solitons. We hope that the present exact (closed-form) solutions may be useful in further numerical analysis of the nonlinear partial differential equation investigated in this work. The availability of computer codes like Mathematica or Maple facilitates the tedious algebraic calculations.

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