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A TIME-DEPENDENT VEHICLE ROUTING PROBLEM SOLVED BY IMPROVED SIMULATED ANNEALING

Narges NOROUZI, Mohsen SADEGH-AMALNICK, Reza TAVAKKOLI-MOGHADDAM

University of Tehran, College of Engineering, School of Industrial Engineering, Tehran, Iran
E-mail: narges.norouzi@ut.ac.ir

In this paper a new multi-objective mathematical model for the vehicle routing problem is presented in which the total travel times and total travel distance are minimized in a time-dependent situation. Moreover, driver satisfaction is maximized based on balancing the distribution of goods in view of the limited vehicle capacities. Since the vehicle routing problems belong to the category of NP-hard problems and exact solutions are not practical in large scales, hence a new method based on the improved simulated annealing algorithm is proposed to obtain efficient solutions with reasonable computational time; and the related results are compared with the particle swarm optimization (PSO) algorithm.

Key words: transportation, time dependent vehicle routing problem, improved meta-heuristic, simulated annealing.

1. INTRODUCTION

A time-dependent vehicle routing problem (TDVRP) include problems in which fleet of vehicles present service from depot to different geographically located customers so that the total travel time is minimized. In urban areas, traffic is varied in different hours of day and the travel time among locations is a function of both the travelled distance and departure time [1]. Despite of the important applications of the TDVRP, the literature related to this problem is scarce. The earliest work was done by Malandraki [2], who formulated the time-dependent traveling salesman problem (TDTSP). Next, Malandraki and Daskin [3] handled some small problems with 10–25 customers and 2–3 time-periods by a GRASP without considering any improvement phase. In their work, the travel time in per time-zone follows a known step function distribution. The main weakness of the TDVRP in the previous studies was that the FIFO (First-In–First-Out) property was not satisfied. Ichoua *et al*. [1] used a piecewise linear travel time function that satisfies the FIFO property. Also, they applied a Tabu search to handle TDVRP with soft time windows to solve Solomon test problems with three time-zones without presenting any mathematical model. Furthermore, Van Woensel et al. [4] proposed a tabu search to work on the TDVRP without time windows. They used a queuing theory to approximate travel speeds with assuming that the estimating flows are much easier than speeds in real-life cases. Donati et al. [5] used an ACO to handle the TDVRP with hard time windows in a network in Padua (Italy). They applied their model for up to 80 customers with 144 time-periods and given distances. Recently, Kok et al. [6] proposed a speed model on real road networks that shows the major elements of the peak hour traffic congestion to generate a set of realistic VRP instances.

In recent years, heuristics [7] and meta-heuristics, such as the tabu search [8], the ant system [9], particle swarm optimization (PSO) [10], and variable neighborhood search (VNS) [11] have been presented. It is proven that VRPs belong to the category of NP-hard problems [12]; therefore the model is solved by an improved simulated annealing (ISA). The related results are compared with the results obtained by the particle swarm optimization (PSO) algorithm. The results show that the ISA performs more efficient than PSO.

The main contributions of this paper can be summarized as follows. Based on the importance of the cost of transportation, the distance travelled and travel time of each route is minimized. Moreover, driver satisfaction is maximized based on balancing the distribution of goods. This model is solved by an improved SA.

The rest of this paper is organized as follows. Section 2 introduces the problem and the model formulation. Problem-solving methodology is described in section 3, and computational results are discussed in Section 4. Finally a conclusion follows in Section 5.

2. PROBLEM DEFINITION

TDVRPs include problems, in which the fleet of vehicles presents service from a depot to a geographically dispersed customers' set with specific demands so that the total traveled time is minimized. The travel time is a function of both the travelled distance and customer's departure time. Because in urban areas, traffic condition varies throughout the day, travel time and speed between two same locations fluctuate in different hours of the day. In fact, the travel speed reduces considerably during the rush hours.

The problem is solved under the following assumptions:

1. Each vehicle has a fixed capacity.
2. The total demands of the customers does not exceed from the capacity of the vehicle.
3. Each customer location is only served by one vehicle and each vehicle services only one route.
4. The fleet is homogenous and the capacities of the vehicles are the same.
5. Each vehicle starts from and terminates at the depot.

Let G = (V,A) be a graph, where $V=\{v_1,v_1,...,v_n\}$ is the nodes set, and $A=\{(v_i,v_j): v_i,v_j \in V\}$ is the arcs set where each arc (v_i, v_j) is associated with a non-negative cost C_{ij}. The time horizon is divided into P time intervals, T_1; T_2; . . . ; T_P and a travel speed matrices $V_T = (\vartheta_{ijp}), p \in [1, P]$ is defined. The objective is to minimize a weighted sum of total travel time and total distance traveled over all customers.

The objective is to minimize a weighted sum of total travel time, total distance traveled and total fuel consumption over all customers. Followings are a number of parameters used in the presented mathematical model.

2.1. Model formulation

n – number of nodes (i.e., customers
nv – total number of vehicles
c_t – unit cost of travel time
c_d – unit cost of travel distance
q_i – number of demands in node i ($i = 1,\ldots,n$)
d_{ij} – distance between node (i,j)
t_{ij}^v – travel time to travel the distance (i,j)
C_v – capacity of vehicle v
s_i – required time for servicing the customer (node) i
ϑ_{ijp} – vehicle speed in time interval $p \in [1, P]$
$\overline{t}_p$ – upper bound of time interval p
s_v – the expected load allocated to vehicle v
$\overline{S}$ – the average of the expected load allocated to the vehicles
T – maximum time that vehicle v can be used
$S = \{i \mid i = 1,\ldots,n\}$ – collection of nodes

Decision variables

We have two decision variables as follows:

$$\begin{cases} x^{v}{}_{ij} = 1, & \text{if vehicle } v \text{ travels through route } (i, j) \\ x^{v}{}_{ij} = 0, & \text{otherwise.} \end{cases}$$

t_i^v – the departure time from node i by vehicle v.

Next, the problem can be mathematically formulated as follows:

$$\text{Minimize } Z = c_t \sum_{i,j\in S} \sum_{v=1}^{nv} \delta_{ij}^{v} x_{ijv} + c_d \sum_{i,j\in S} \sum_{v=1}^{nv} d_{ij} x_{ijv} + \\ + \text{Minimize } Z = \sum_{v=1}^{nv} \left| s_v - \overline{S} \right| \tag{1}$$

$$\sum_{i=0}^{n} \sum_{v=1}^{nv} x_{ijv} = 1 \quad ; \ j = 1, 2, \ldots, n \tag{2}$$

$$\sum_{j=0}^{n} \sum_{v=1}^{nv} x_{ijv} = 1 \quad ; \ i = 1, 2, \ldots, n \tag{3}$$

$$\sum_{i=0}^{n} x_{imv} - \sum_{j=1}^{n} x_{mjv} = 0 \quad ; \ m = 1, 2, \ldots, n \quad ; v = 1, 2, \ldots, nv \tag{4}$$

$$t_{ij}^{v} = \sum_{(i,j)\in S} \sum_{p=1}^{P} \left(\left(\frac{d_{ij}}{\vartheta_{ijp}} - \overline{t_p} \right) \times \frac{\vartheta_{ijp}}{\vartheta_{ijp+1}} + \overline{t_p} - t_j^{v} \right) x_{ijv} \ ; \quad v = 1, 2, \ldots, nv \tag{5}$$

$$t_j^{v} = \sum_{i=1}^{n} \sum_{v=1}^{nv} \sum_{p=1}^{P} t_i^{v} x_{ijv} + \sum_{i=1}^{n} \sum_{v=1}^{nv} t_{ij}^{v} + s_j^{v} \quad ; \quad j = 1, \ldots, n \tag{6}$$

$$\sum_{i=0}^{n} s_i \sum_{j=1}^{n} x_{ijv} + \sum_{i=0}^{n} \sum_{j=1}^{n} t_{ij}^{v} x_{ijv} \le T \quad ; \ v = 1, 2, \ldots, nv \ , p = 1, \ldots, P \tag{7}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{n} q_i x_{ijv} \le C_v \quad ; \ v = 1, 2, \ldots, nv \tag{8}$$

$$\sum_{i=1}^{n} x_{i0v} \le 1 \quad ; \ v = 1, 2, \ldots, nv \tag{9}$$

$$\sum_{j=1}^{n} x_{0jv} \le 1 \quad ; \ v = 1, 2, \ldots, nv \tag{10}$$

$$\overline{S} = \frac{1}{nv} \sum_{v=1}^{nv} s_v \tag{11}$$

$$\sum_{v=1}^{nv}\sum_{j\in S}\sum_{j\notin S} x_{ijv} \leq |S| - r(S) \qquad \forall S \subseteq A - \{1\} \qquad S \neq \varphi \tag{12}$$

$$x_{ij}^{v} \in S, x_{ij}^{v} \in [0.1]. \tag{13}$$

In this formulation, the objective function (1) is divided into three terms. The first term minimizes the total travel time, the second term minimizes the total distance traveled and third term balancing the goods distributed to vehicles according to their capacities. Constraints (2) and (3) ensure that each customer location is serviced by only one vehicle. Constraint (4) states that if a vehicle arrives at a node (i.e., customer location), it must leave it and by this way the route continuity is ensured. Constraint (5) calculates the total time for traveling between customers i and j by vehicle v in the different time periods with changing in travel speed with satisfying FIFO property. Constraint (6) states time for leaving the jth customer in the time period p. Constraint (7) ensures that the sum of the required time for servicing customer i and the required time for traveling between customers i and j in different periods do not have to exceed from the maximum time, in which vehicle v can be used. Also, Constraint (8) imposes that the vehicle capacity does not exceed from its limit. Constraints (9) and (10) impose that a depot is the first and final destination of each vehicle. The average of the expected load allocated to the vehicles is introduced in constraint (12) and finally Constraint (13) eliminates the sub-tours.

3. PROBLEM-SOLVING APPROACH

Problem solving methodology is presented based on improved simulated annealing in section 3.1. In section 3.2 an initial solution generation is presented and then some of the neighborhood search methods for improving solutions are proposed in section 3.3.

3.1. Improved simulated annealing

Simulated annealing (SA) is the process of physical annealing with solids, in which a crystalline solid is heated and then allowed to cool very slowly until it achieves its most regular possible crystal lattice configuration (i.e., its minimum lattice energy state), and thus is free of crystal defects. If the cooling schedule is sufficiently slow, the final configuration results in a solid with such superior structural integrity. Simulated annealing establishes the connection between this type of thermo-dynamic behavior and the search for global minima for a discrete optimization problem. Furthermore, it provides an algorithmic means for exploiting such a connection [13-14].

The SA parameters are as follow:

Epoch length (EL) = Number of accepted solutions in each temperature for achieving to equilibrium

MTT = Maximum number of consecutive temperature trails.

T_0 = Initial temperature.

α = Rate of the current temperature decrease (cooling schedule).

X = A feasible solution.

$C(X)$ = Objective function value of solution X.

L = Counter for the number of accepted solutions in each temperature;

K = Counter for the number of consecutive temperature trails, where T_K is equal to temperature in iteration K.

3.2. Initial solution generation

Generating the initial solution is the first step in any meta-heuristics that influences on the quality of the best solution obtained from them. To generate initial solutions of the modified SA, a creative approach is applied in which both travel speed and distance are simultaneously considered as follows:

1) Sort the customers based on descending travel speed (*i.e.*, the customer with higher travel speed is served earlier).

Repeat below steps for count = 1: popsize.

2) Create a new route: Allocate the first *m* customers to the *nv* vehicle. Allocate *nv* new vehicle (*i.e.*, *v*) with maximum capacity to those from depot.
3) Determine neighborhood nodes: find the un-served nearest *k* nodes neighborhoods (*k* is the neighborhood size). Then select that node with the highest travel speed and served it such that if vehicle *v* served selected node, the time and capacity constraint of the vehicle are not violated. If such node is found then vehicle *v* services *k* after $r(x_{rkv} = 1)$. Otherwise select other *k* neighborhoods and try to allocate them to vehicle *v*.
4) Complete the route: Repeat step 3 till vehicle *v* returns to depot.
5) Complete the service: Repeat above steps till all of the nodes are served.
6) If all of the customers are allocated, count = count+1, otherwise repeat steps 2 to 5.

Using this initial solution method, vehicles can select routes with higher travel speeds and lower distance.

3.3. Neighborhood improvement methods

Two efficient genetic operators are designed for searching in feasible solution space and obtaining neighborhood solution as follows:

Random method. Select randomly a route then select as the max [0.1 (*i.e.*, length of the route), 2] nodes in each route, and change the integer number of the selected nodes in the bound of [0, *m*] randomly (to change the vehicle that services the selected node). In other words, a node is departed from one route and it is added to another route using insertion heuristics. New, insertion is accepted if the vehicle capacity and maximum time that vehicle *v* can be used are not violated.

2-opt exchange operator. The 2-opt operator improves single route and exchanges the route direction between two consecutive nodes. If the cost function of the route has been improved, then modified route is kept; otherwise the route returns to the last condition. Fig. 1 shows the 2-opt exchange operation in which the edge (i, i+1) and (j, j+1) are replaced by edge (i, j) and (i+1, j+1) and reversing the direction of customers between i+1 and j.

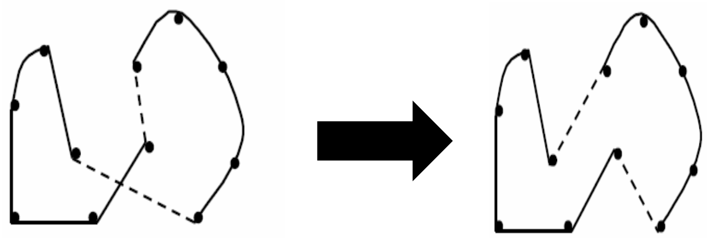

Fig 1. – 2-opt exchange operation.

The main steps of the proposed improved SA embedded with improvement operators are as follows.

Step 1: Generate an initial solution according to the initial solution generation algorithm.

Step 2: Generate a neighborhood solution for X_1 (*i.e.*, X_2) and estimate its corresponding objective function value, E_2.

Step 3: if $E_1 >= E_2$ then X_2 is a new solution of the given problem, else regenerate a random number, Rnd (0, 1).

Step 4: if Rnd (0, 1) < exp $\{-(E_2 - E_1)/\mathrm{KT}\}$, then accept X_2; else, reject X_2 and new solution is X_1. Also *k* is a constant and it considers 1 in problems.

Step 5: Repeat the algorithm several times for *T* temperature.

Step 6: Decrease *T* by a relation such as $(n + 1) = \eta \times T(n)$, $0 < \eta < 1$, in which η is cooling schedule.

Step 7: Terminate the algorithm when the convergence criterion is satisfied.

4. COMPUTATIONAL RESULTS

The computational results are obtained from the proposed solution approach in this section. All solution procedures are coded in the visual basic, and all test problems are run using the Intel Core i3, 1.66 GHz compiler and 4GB of RAM. To solve this problem by the proposed improved SA, the related parameters are set as EL = 100, MTT = 100, T_0 = 150 and η = 0.99.

Also, the performance of the proposed algorithms is tested on the set of 56 Solomon's test cases [15]. In these test problems and for all of the test problems in the original data, there are 100 customers that are divided into six sets of R1, R2, C1, C2, RC1 and RC2. Because of the difference between the proposed problem and the classical VRP, some assumptions are inserted into the Solomon's test problems for adjusting the standard test problems with the proposed problem. The number of vehicles was set to the number of routes in the best solution reported in the literature for each problem. Also, *cf* and *ct* are chosen from uniform bounds (1.25, 1.95) and Uniform (0.35, 0.65) respectively. To illustrate the time dependency of the problem, we consider 2 different of road links (city, and highway links), which imply the various speed levels in the different hours of the day. Generally, the road type and the time of the day to start service vary travel speeds. Also, the time horizon is divided into 5 periods from 7:00 AM to 5:00 PM and the corresponding travel speeds for each road type are chosen from uniform bounds (40, 60) for city arcs and Uniform (55, 90) for highway links respectively. Travel speed distributions are assigned randomly to customers' arcs in which categories 1, 2 are selected with the same probability.

The objective function is divided into three terms that minimizes the total travel time, the total travel distance and maximizes driver satisfaction based on balancing the distribution of goods in view of the limited vehicle capacities, respectively. In each of the test problems, two objective functions are changed into one objective function by weighting each term. The weights of the objective terms are considered 0.3, 0.5, and 0.2 respectively.

Table 1

Comparison of the performances of the proposed algorithms

	PSO		ISA		
	Objective	CPU Time	Objective	CPU Time	Improvement
C1	2929.69	553.8	2755.69	583.8	0.12
C2	2235.93	573.36	2474.93	614.36	0.09
R1	3474.78	621.9	3885.78	670.9	0.15
R2	3594.94	666.86	3594.94	690.86	0.1
RC1	3819.92	658.23	4088.92	690.23	0.08
RC2	3707.79	776.92	3870.79	821.92	0.17
Average	3071.51	641.85	2899.33	678.85	0.11

Table 1 compares the performances produced by the proposed algorithms. The algorithm was ran three times, and an average result is found for each problem. In this table, the first column is the problem name. The second and third columns demonstrate obtained solution and the run time by PSO and the columns fourth and fifth are related to those obtained by ISA. Also, column six is associated to the ISA improvement. The results obtained with improved SA perform better than those obtained with PSO. In fact, the results obtained by ISA show an improvement in average of test problems by 11%. The average run time of ISA and

PSO in three sets of C, R and RC are 641.85 and 678.85 respectively. Therefore, the proposed ISA is more convenient in quality of the solution but PSO is better in running time. Also, convergence rate for Problem C108 is shown in Fig. 2.

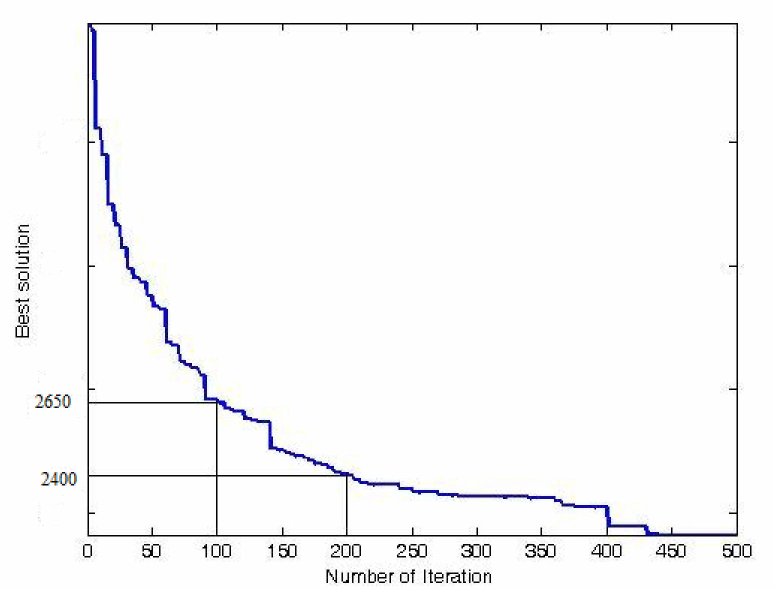

Fig. 2 – Convergence rate for Problem C201.

5. CONCLUSION

In this paper, an improved simulated annealing algorithm was proposed to solve a time-dependent vehicle routing problems (TDVRP). In the TDVRP, travel time is a function of both the travelled distance and customer’s departure time. Because in urban areas, traffic condition varies throughout the day, travel time and speed between two same locations fluctuate in different hours of the day. In fact, the travel speed reduces considerably during the rush hours. Based on the importance of the cost of transportation, the distance travelled and travel time of each route is minimized. Also, driver satisfaction is maximized based on balancing the distribution of goods. The results have been compared with the results obtained by PSO. The results showed that our proposed ISA was an efficient algorithm for solving the TDVRP with a reasonable computational cost. In fact, the results obtained by ISA show an improvement in average of test problems by 11%. The results show the PSO is convenient when time is restricted for solving problem and ISA is better when the quality of the solution is more important.

REFERENCES

1. Ichoua, S., Gendreau, M., Potvin, JY. *Vehicle dispatching with time-dependent travel times*. European Journal of Operational Research, **144**, *2*, pp. 379–396, 2003.
2. Malandraki, C. *Time dependent vehicle routing problems: Formulations, solution algorithms and computations experiments.* Ph.D. Dissertation, Northwestern University, Evanston, III, 1989.
3. Malandraki, C., Daskin, MS. *Time dependent vehicle routing problems: formulations, properties and heuristic algorithms.* Transport Science, **26**, *3*, pp. 185–200, 1992.
4. Van Woensel, T., Kerbache, L., Peremans, H., Vandaele, N. *Vehicle routing with dynamic travel times: A queueing approach.* European journal of operational research, **186**, *3*, pp. 990–1007, 2008.
5. Donati, A.V., Montemanni, R., Casagrande, N., Rizzoli A.E., Gambardella, L.M. *Time dependent vehicle routing problem with a multi ant colony system*, European Journal of Operational Research, **185**, *3*, pp. 1174–1191, 2008.
6. Kok, A.L., Hans, E.W., Schutten, J.M.J. *Vehicle routing under time dependent travel times: The impact of congestion avoidance*. Computers & Operations Research, **39**, pp. 910–918, 2012.
7. Sariklis D, Powell, S., A heuristic method for the open vehicle routing problem. J. of the Operational Research Society, **51**, pp. 564–573, 2000.

8. Jia, H., Li, Y., Dong, B., Ya, H., *An Improved Tabu Search Approach to Vehicle Routing Problem*. Procedia – Social and Behavioral Sciences, **96**, pp. 1208–1217, 2013.
9. Gajpal, Y., Abad, P., *An ant colony system (ACS) for vehicle routing problem with simultaneous delivery and pickup. Computers & Operations Research*, **36**, *12*, pp. 3215–3223, 2009.
10. Goksal, F.P., Karaoglan, I., Altiparmak, F., *A hybrid discrete particle swarm optimization for vehicle routing problem with simultaneous pickup and delivery*. Computers & Industrial Engineering, **65**, *1*, pp. 39–53, 2013.
11. Kuo, Y., Wang, C-C. *A variable neighborhood search for the multi-depot vehicle routing problem with loading cost*. Expert Systems with Applications, **39**, *8*, pp. 6949–6954, 2012.
12. Lenstra, J.K., Rinnooy Kan, A.H.G., *Complexity of vehicle and scheduling problem*. Networks, **11**, pp. 221–227.
13. Kirkputrick, F., Gelatt, C., Vecci, M., *Optimization by simulated annealing*, Science, **2**, pp. 117–133, 1981.
14. Tavakkoli-Moghaddam, R., Alinaghian, M., Salamatbakhsh, A., Norouzi, N. *A hybrid meta-heuristic algorithm for the vehicle routing problem with stochastic travel times considering the driver's satisfaction*. Journal of Industrial Engineering International, **8**, *4*, pp.1–12, 2012.
15. Solomon MM., *Algorithms for the vehicle routing and scheduling problem with time windows constraints*. Operations Research, **35**, pp. 254–265, 1987.

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