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SOLITARY WAVES AND BIFURCATION ANALYSIS OF THE K(M, N) EQUATION WITH GENERALIZED EVOLUTION TERM

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Abstract. The bifurcation analysis of the $K(m, n)$ equation, which serves as a generalized model for the Korteweg-de Vries equation describing the dynamics of shallow water waves on ocean beaches and lake shores, is carried out in this paper. The phase portraits are given and solitary wave solutions are obtained. Singular periodic wave solutions are also given in this work.

Key words: solitary waves; singular periodic solutions; bifurcation analysis.

1. INRODUCTION

The dynamics of shallow water waves is a very active area of research in applied fluid mechanics. There are several models that describe this dynamics. They are Korteweg-de Vries (KdV) equation, modified KdV equation, Gardner equation, Kawahara equation, Peregrine equation or regularized long wave (RLW) equation, Boussinesq equation, Kadomtsev-Petviashvili equation and several others. While these models describe single layered shallow water dynamics, there is another set of models that govern the dynamics of two-layered flows. A few such models are Bona-Chen equation, Gear-Grimshaw equation, coupled KdV equation and others. Many mathematical principles and techniques were applied to these nonlinear evolution equations in the past [1–15] and a series of fast computational techniques were recently reported to solve both linear and nonlinear partial differential equations, including fractional partial differential equations; see e.g. Refs. [16–21].

This paper stays focused on studying a generalized version of the KdV equation that was introduced a few decades ago by Rosenau and Hyman. This is referred to the so-called $K(m, n)$ equation. This model was extensively studied by several authors. Later several other models were proposed as an extended version. These are Rosenau-Kawahara and Rosenau-KdV equations. The latter equation was further extended to Rosenau-Kawahara-RLW equation. A plethora of results can be found in the literature pertaining to the above mentioned nonlinear evolution equations and other nonlinear partial differential equations that find direct applications in diverse areas of nonlinear science; see e.g. Refs. [22–39]. The rest of this paper is organized as follows. In Sec. 2 we briefly introduce the governing mathematical model for the $K(m,n)$ equation with generalized evolution term. Section 3 is devoted to the phase portrait analysis based on the qualitative theory of the dynamical system under consideration. Different solitary wave solutions, including singular periodic wave solutions are given in Sec. 4. Finally, the conclusions and some possible extensions of this study are given in Sec. 5.

2. MATHEMATICAL MODEL

The $K(m,n)$ equation with generalized evolution term [1, 2, 8] takes the following form:

$$\left(q^{l}\right)_{t} + aq^{m}q + b\left(q^{n}\right)_{xxx} = 0, \tag{1}$$

where the first term is the generalized evolution term, while the second term accounts for the nonlinearity. The third term is for the generalized dispersion. Also, a and b are arbitrary constants, while l, m, and $n \in Z^+$. Equation (1) reduces to the $K(m, n)$ equation for $l = 1$. Therefore, for $l = 1$, $K(1,1)$ is the KdV equation, while $K(2,1)$ is the mKdV equation. Both the KdV and mKdV equations govern the dynamics of shallow water waves and these equations were extensively studied in the literature of fluid dynamics. These equations also arise in plasma physics and consequently these dynamical models are visible in many works on plasma physics. In this paper we will carry out the bifurcation analysis of Eq. (1) and this issue and a few other problems will be discussed in the next sections of this paper.

3. PHASE PORTRAITS AND QUALITATIVE ANALYSIS

We assume that traveling wave solutions of Eq. (1) take the form

$$q(x,t) = U(\xi), \quad \xi = x - ct, \tag{2}$$

where c is the wave speed. By replacing Eq. (2) into Eq. (1), one recovers

$$-c\left(U^l\right)' + aU^m U' + b\left(U^n\right)''' = 0, \tag{3}$$

where the prime denotes the derivative with respect to ξ. Equation (3) is then integrated term by term one time, where the integration constant is considered to be zero. This converts Eq. (3) into

$$-cU^l + \frac{a}{m+1}U^{m+1} + b\left(U^n\right)'' = 0. \tag{4}$$

Let $l = n$, then we use the transformation

$$U(\xi) = \varphi^{\frac{1}{n}}(\xi) \tag{5}$$

that will reduce Eq. (4) into the ordinary differential equation (ODE)

$$-c\varphi + \frac{a}{m+1}\varphi^{\frac{m+1}{n}} + b\varphi'' = 0. \tag{6}$$

To initiate the analysis, we introduce the notation

$$\theta = \frac{c}{b}, \quad \delta = \frac{a}{b(m+1)}. \tag{7}$$

Letting $\varphi' = z$ then we get the following coupled system of coupled differential equations:

$$\begin{aligned} \frac{\mathrm{d}\varphi}{\mathrm{d}\xi} &= z \\ \frac{\mathrm{d}z}{\mathrm{d}\xi} &= \theta\varphi - \delta\varphi^{\frac{m+1}{n}}. \end{aligned} \tag{8}$$

Obviously, the above system (8) is a Hamiltonian one with the Hamiltonian function:

$$H(\varphi, z) = z^2 - \theta\varphi^2 + \frac{2n\delta}{m+1+n}\varphi^{\frac{m+1+n}{n}}. \tag{9}$$

In order to investigate the phase portraits of the coupled system (8), we set

$$f(\varphi) = \theta\varphi - \delta\varphi^{\frac{m+1}{n}}. \tag{10}$$

Obviously, when $\theta\delta > 0$, $f(\varphi)$ has three zero points, φ_-, φ_0, and φ_+ that are given as follows:

$$\varphi_- = -\left(\frac{\theta}{\delta}\right)^{\frac{n}{m+1-n}}, \quad \varphi_0 = 0, \quad \varphi_+ = \left(\frac{\theta}{\delta}\right)^{\frac{n}{m+1-n}}. \tag{11}$$

When $\theta\delta \le 0$, $f(\varphi)$ has only one zero point

$$\varphi_0 = 0. \tag{12}$$

Letting $(\varphi_l, 0)$ be one of the singular points of the system (8), then the *characteristic values* of the linearized system corresponding to Eqs. (8) at the singular points $(\varphi_l, 0)$ are

$$\lambda_\pm = \pm\sqrt{f'(\varphi_l)}. \tag{13}$$

From the qualitative theory of dynamical systems, we know the following:

(**I**) If $f'(\varphi_l) > 0$, $(\varphi_l, 0)$ is a saddle point.

(**II**) If $f'(\varphi_l) < 0$, $(\varphi_l, 0)$ is a center point.

(**III**) If $f'(\varphi_l) = 0$, $(\varphi_l, 0)$ is a degenerate saddle point.

Therefore, in Fig. 1 we show the bifurcation phase portraits of the system (8). Let

$$H(\varphi, z) = h, \tag{14}$$

where *h* is the Hamiltonian.

Next, we consider the relations between the orbits of (8) and the Hamiltonian *h*. Therefore, we set

$$h^* = \left|H(\varphi_+, 0)\right| = \left|H(\varphi_-, 0)\right|. \tag{15}$$

According to Fig. 1, we have the following propositions; see also Refs. [5, 11–15]:

PROPOSITION 1. *Suppose that* $\theta > 0$ *and* $\delta > 0$, *then one has the following*:

(**I**) *When* $h \le -h^*$, *the system (8) does not have any closed orbits*.

(**II**) *When* $-h^* < h < 0$, *the system (8) has two periodic orbits* Γ_1 *and* Γ_2.

(**III**) *When* $h = 0$, *the system (8) has two homoclinic orbits* Γ_3 *and* Γ_4.

(**IV**) *When* $h > 0$, *the system (8) has a periodic orbit* Γ_5.

PROPOSITION 2. *Suppose that* $\theta < 0$ *and* $\delta < 0$, *then one has the following*:

(**I**) *When* $h < 0$ or $h > h^*$, *the system (8) does not have any closed orbits*.

(**II**) *When* $0 < h < h^*$, *the system (8) has three periodic orbits* Γ_6, Γ_7, *and* Γ_8.

(**III**) *When* $h = 0$, *the system (8) has two periodic orbits* Γ_9 *and* Γ_{10}.

(**IV**) *When* $h = h^*$, *the system (8) has two heteroclinic orbits* Γ_{11} *and* Γ_{12}.

PROPOSITION 3. (**I**) *When* $\theta \ge 0, \delta > 0$ *and* $h > 0$, *the system (8) has periodic orbits*.

(**II**) *When* $\theta \le 0, \delta < 0$, *the system (8) does not have any closed orbits*.

From the qualitative theory of dynamical systems, we know that a smooth *solitary wave solution* of a partial differential system corresponds to a smooth homoclinic orbit of the wave equation. A smooth *kink wave solution* or a unbounded wave solution corresponds to a smooth heteroclinic orbit of the wave equation. Similarly, a periodic orbit corresponds to a *periodic traveling wave solution* of a partial differential system. According to the above analysis, we have the following propositions.

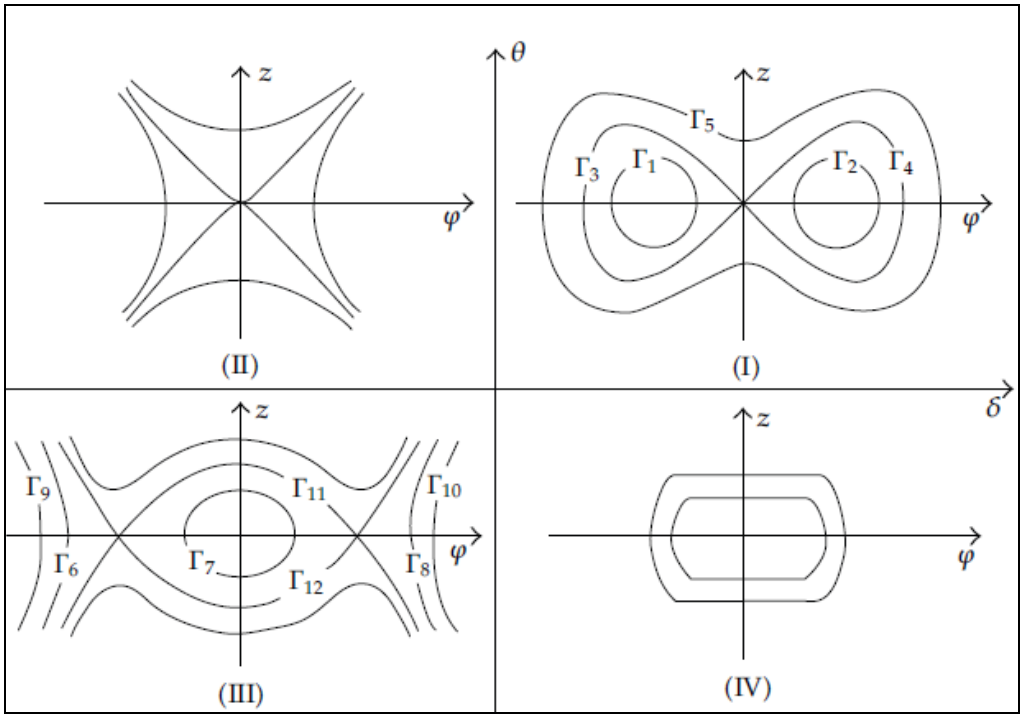

Fig. 1. – Variations of z with respect to φ.

PROPOSITION 4. *If* $\theta > 0$ *and* $\delta > 0$, *one has the following*:

(**I**) *When* $-h^* < h < 0$, *Eq. (1) has two periodic wave solutions (corresponding to the periodic orbits* Γ_1 *and* Γ_2*).*

(**II**) *When* $h = 0$, *Eq. (1) has two solitary wave solutions (corresponding to the periodic orbits* Γ_3 *and* Γ_4 *in Fig. 1).*

(**III**) *When* $h > 0$, *Eq. (1) has two periodic wave solutions (corresponding to the periodic orbit* Γ_5 *in Fig. 1).*

PROPOSITION 5. *If* $\theta < 0$ *and* $\delta < 0$, *one has the following*:

(**I**) *When* $0 < h < h^*$, Eq. *(1) has two periodic wave solutions (corresponding to the periodic orbit* Γ_7 *in Fig. 1).*

(**II**) *When* $h = 0$, *Eq. (1) has periodic blow-up wave solutions (corresponding to the periodic orbits* Γ_6 *and* Γ_8 *in Fig. 1).*

(**III**) *When* $h = h^*$, *Eq. (1) has two kink-type solitary wave solutions (corresponding to the heteroclinic orbits* Γ_{11} *and* Γ_{12} *in Fig. 1*).

4. EXACT TRAVELING WAVE SOLUTIONS

First, we will obtain the explicit expressions of *traveling wave solutions* for Eq. (1) when $\theta > 0$ and $\delta > 0$. From the phase portraits, we see that there are two symmetric homoclinic orbits Γ_3 and Γ_4 connected at the saddle point (0, 0). In the (φ, z)-plane the expressions of the homoclinic orbits are given as [3, 11–15]

$$z = \pm\sqrt{\frac{2n\delta}{m+1+n}}\varphi\sqrt{-\varphi^{\frac{m+1-n}{n}} + \frac{(m+1+n)\theta}{2n\delta}}. \tag{16}$$

Substituting (16) into $d\varphi / d\xi = z$ and integrating them along the orbits Γ_3 and Γ_4, we have

$$\pm\int_{\varphi_1}^{\varphi}\frac{1}{s\sqrt{-s^{\frac{m+1-n}{n}}+\frac{(m+1+n)\theta}{2n\delta}}}\mathrm{d}s=\sqrt{\frac{2n\delta}{m+1+n}}\int_0^{\xi}\mathrm{d}s$$
$$\pm\int_{\varphi_2}^{\varphi}\frac{1}{s\sqrt{-s^{\frac{m+1-n}{n}}+\frac{(m+1+n)\theta}{2n\delta}}}\mathrm{d}s=\sqrt{\frac{2n\delta}{m+1+n}}\int_0^{\xi}\mathrm{d}s, \tag{17}$$

where $\varphi_1=-\left(\frac{(m+1+n)\theta}{2n\delta}\right)^{\frac{n}{m+1-n}}$ and $\varphi_1=\left(\frac{(m+1+n)\theta}{2n\delta}\right)^{\frac{n}{m+1-n}}$.

Computing the above integrals give

$$\varphi=\left\{\sqrt{\frac{(m+1+n)\theta}{2n\delta}}\operatorname{sech}\left(\frac{m+1-n}{2n}\sqrt{\theta}\xi\right)\right\}^{\frac{2n}{m+1-n}}$$
$$\varphi=-\left\{\sqrt{\frac{(m+1+n)\theta}{2n\delta}}\operatorname{sech}\left(\frac{m+1-n}{2n}\sqrt{\theta}\xi\right)\right\}^{\frac{2n}{m+1-n}}. \tag{18}$$

Noting the Eqs. (2), (5), and (7), we get the following solitary wave solution:

$$q(x,t)=\pm\left\{\sqrt{\frac{(m+1+n)(m+1)c}{2na}}\operatorname{sech}\left(\frac{m+1-n}{2n}\sqrt{\frac{c}{b}}(x-ct)\right)\right\}^{\frac{2}{m+1-n}}. \tag{19}$$

Second, we will obtain the explicit expressions of traveling wave solutions for Eq. (1) when $\theta<0$ and $\delta<0$. From the phase portraits, we note that there are two special orbits Γ_9 and Γ_{10}, which have the same Hamiltonian with that of the center point (0,0). In the (φ,z)-plane the expressions of the orbits are given as

$$z=\pm\sqrt{-\frac{2n\delta}{m+1+n}}\varphi\sqrt{\varphi^{\frac{m+1-n}{n}}-\frac{(m+1+n)\theta}{2n\delta}}. \tag{20}$$

Substituting Eqs. (20) into $\mathrm{d}\varphi/\mathrm{d}\xi=z$ and integrating them along the orbits Γ_9 and Γ_{10}, we have

$$\pm\int_{\varphi}^{+\infty}\frac{1}{s\sqrt{s^{\frac{m+1-n}{n}}-\frac{(m+1+n)\theta}{2n\delta}}}\mathrm{d}s=\sqrt{-\frac{2n\delta}{m+1+n}}\int_0^{\xi}\mathrm{d}s,$$
$$\pm\int_{\varphi_4}^{\varphi}\frac{1}{s\sqrt{s^{\frac{m+1-n}{n}}-\frac{(m+1+n)\theta}{2n\delta}}}\mathrm{d}s=\sqrt{-\frac{2n\delta}{m+1+n}}\int_0^{\xi}\mathrm{d}s, \tag{21}$$

where $\varphi_4=\left(\frac{(m+1+n)\theta}{2n\delta}\right)^{\frac{n}{m+1-n}}$. Computing the above integrals leads to

$$\varphi=\pm\left\{\sqrt{\frac{(m+1+n)\theta}{2n\delta}}\sec\left(\frac{m+1-n}{2n}\sqrt{-\theta}\xi\right)\right\}^{\frac{2n}{m+1-n}},$$
$$\varphi=\pm\left\{\sqrt{\frac{(m+1+n)\theta}{2n\delta}}\csc\left(\frac{m+1-n}{2n}\sqrt{-\theta}\xi\right)\right\}^{\frac{2n}{m+1-n}}. \tag{22}$$

Noting the Eqs. (2), (5), and (7), one recovers the following *singular periodic solutions*:

$$q(x,t) = \pm\left\{\sqrt{\frac{(m+1+n)(m+1)c}{2na}}\sec\left(\frac{m+1-n}{2n}\sqrt{-\frac{c}{b}}(x-ct)\right)\right\}^{\frac{2}{m+1-n}} \tag{23}$$

and

$$q(x,t) = \pm\left\{\sqrt{\frac{(m+1+n)(m+1)c}{2na}}\csc\left(\frac{m+1-n}{2n}\sqrt{-\frac{c}{b}}(x-ct)\right)\right\}^{\frac{2}{m+1-n}}. \tag{24}$$

The conditions for these singular periodic solutions to exist are $\theta < 0$ and $\delta < 0$, which amounts to saying

$$bc < 0. \tag{25}$$

This condition serves as a constraint for the existence of such solutions.

5. CONCLUSIONS

This paper carried out the bifurcation analysis of the generalized version of the Korteweg-de Vries equation that is also referred to as the $K(m, n)$ equation. The phase portraits are displayed and the analysis resulted in the retrieval of solitary wave solutions. As a byproduct, singular periodic solutions fell out of the analysis although these solutions have no direct implications in the area of fluid dynamics. While this paper is from a purely analytical standpoint, a numerical analysis of the equation will be carried out in future and a detailed analysis will be performed with the aid of some of the computational techniques that were recently reported [16–21]. This will lead to a separate work that will be published elsewhere.

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