INFLUENCE OF KINEMATIC PARAMETERS AND TOOTH GEOMETRY ON GEAR TOOTH ROOT LOAD CAPACITY

Mileta RISTIVOJEVIĆ, Tatjana LAZOVIĆ

University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Belgrade, Serbia
Corresponding author: Tatjana Lazović, E-mail: `tlazovic@mas.bg.ac.rs`

Abstract. Spur gear load carrying capacity and mass are primarily dependent of the tooth root stress. Geometric and kinematic parameters of gear teeth pairs produce multiple effects on the tooth root stress: through tooth root critical section location and root fillet radius, tooth profile shape and load leading point location on tooth profile, as well as through tooth stiffness. In the conventional calculation procedure, according to ISO standard, the influence of geometric and kinematic parameters of the gear pair on the tooth root stress is considered through the "contact ratio factor" Y_ε, whose values are determined using the approximate expressions. To view the real effects of geometric and kinematic parameters of the gear pair on the tooth root stress analytical model is developed, defining non-dimensional relative stress factor. The analysis comprises two groups of cylindrical gear pairs with different gear ratios and different coefficients of addendum modification. The results obtained are compared with the results of standard calculation procedure according to ISO standard. In addition, the developed analytical model is used to explain the assumptions, physicality and boundary conditions of the "contact ratio factor" Y_ε.

Key words: spur gears, tooth root stress, tooth root stress, contact ratio factor, relative stress factor.

1. INTRODUCTION

Power transmission gears are widely used in all areas of engineering. In transmitting power they generate noise, vibration and energy loss. Accordingly, they considerably affect the quality of machines they are installed in. They also affect machines' reliability, energy and ecological efficiency. One of the most significant criteria for assessing operational ability and reliability of gear teeth pairs is load carrying capacity. Teeth pairs' failures due to tooth failure almost always cause damage in other gear components and/or components of machines they are installed in. Hence, teeth pairs are the subject of extensive theoretical and experimental investigations. In general, the goal of investigations on teeth pairs was to make mathematical models to use them for defining operational and critical state of teeth root as much accurately as possible [1–10]. Another trend of investigation is related to finding better suited tooth profile shape [11–13], materials and manufacturing procedure [14, 15] in order to increase teeth load carrying capacity.

Effects of multiple meshes, accuracy of manufacturing and tooth body form on spur gear root and flank stresses were analyzed in paper [11] by applying the FEM. By changing tooth addendum, the number of simultaneously meshed teeth pairs was changed. The carrying capacity of the involute spur gears with asymmetric teeth is investigated in paper [12]. More accurate mathematical models for determining the tooth form factor in spur and helical gears are presented in paper [3]. Paper [4] analyzes boundary values of $K_{F\alpha}$ factor that correspond to even and explicitly uneven load distribution in simultaneously meshed teeth pairs. Influence of teeth size in module domain, below 5 mm, on spur gear tooth root and flank load carrying capacity is analyzed in paper [15]. Experimental investigations demonstrated that tooth root load carrying capacity for the module of 0.6 mm is 40% higher than the value determined according to [1, 2]. Influence of meshed teeth contact line helical angle on helical gear tooth root is analyzed in paper [9]. Based on experimental investigations, it is shown that helical gear pairs with low contact ratio and with increasing helix angle are less advantageous than expected, according to [1, 2]. The influence of base pitch difference sign of simultaneously meshed teeth pairs, load distribution, kinematic parameters and tooth geometry on load carrying capacity of tooth flanks is analyzed

in paper [16]. In paper [16] it is shown that kinematic and geometric parameters of gear pair have strong influence on the choice of stress authoritative in determining the load carrying capacity of spur gear tooth flanks. The standard calculation according to [1, 2] is widely used. However, the proposed procedures for determining some impact factors do not have sufficient accuracy level, as shown in papers [4, 7, 9, 15]. Also, impact factors such as $K_{F\alpha}$ and Y_ε, have not been fully explained in [1, 2] and their values are determined based on an approximate expression. According to the conventional calculation procedure in [2], the influence of geometric and kinematic parameters of the gear pair on the tooth root stress is taken into account through the "contact ratio factor", whose values are determined by applying the approximate expressions. The scientific and professional literature, to the best of authors' knowledge, does not provide the procedure of obtaining the approximate expression for the "contact ratio factor", nor does it provide the assumptions following the obtained approximate expressions. Also, the exact expressions for determining this factor are not given either. Accordingly, there are no conditions for establishing the size of error made by applying the approximate expression when determining the "contact ratio factor". According to the conventional calculation procedure [1, 2], the "contact ratio factor" Y_ε is determined for the meshed gear teeth. It has been shown here that 'in reality' for each gear teeth pair the corresponding factors $Y_{\varepsilon1}$ and $Y_{\varepsilon2}$ should be determined. Physical meaning, definition, assumptions and methodology of determining this factor are presented in this paper.

2. INFLUENCE OF TEETH GEOMETRY AND LOAD DISTRIBUTION ON TOOTH ROOT STRESS

To analyze the stress conditions in the gear tooth root, the tooth itself is approximated with a console shaped mechanical model, on the end of which the load acts in the direction of the teeth profile pressure line. On the basis of [2] and the analysis carried out in [4], the general expression for the tooth root stress, for any contact point $y \in (A...E)$ on meshed teeth profiles (Fig. 1), can be written in the form:

$$\sigma_{Fy} = \sigma_0 K_{\alpha y} Y_{Fy} Y_{Sy} .$$ (1)

The stress σ_0 from Eq. (1) is determined based on known expression:

$$\sigma_0 = \frac{KF_t}{bm} = \frac{K_A K_v K_{F\beta} F_t}{bm} .$$
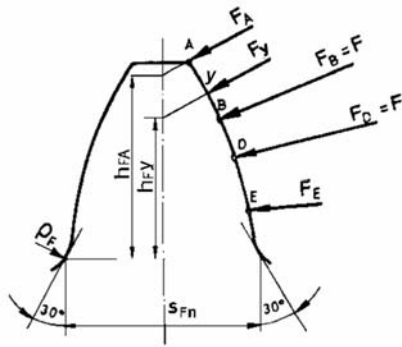

Fig. 1 – Critical section, bending moment arm and contact points within which the load is transferred.

As the contact point moves along the contact line of the tooth profile, the tooth root stress changes during the contact period, due to the load and tooth geometry changes. In order to determine the appropriate stress for evaluation of the tooth root strength, the manner in which the stress changes in the tooth root, during the contact period, should be known. According to Eq. (1), the change in the tooth root stress depends on the character of change in the load distribution factor $K_{\alpha y}$, tooth form factor Y_{Fy} and stress correction factor Y_{Sy}. The analysis of stress change in the tooth root throughout the contact period has been carried out taking into account the relation between the tooth root stress for any contact point and contact point A as follows:

$$\frac{\sigma_{Fy}}{\sigma_{FA}} = \frac{Y_{Fy} Y_{Sy}}{Y_{FA} Y_{SA}} \cdot \frac{K_{\alpha y}}{K_{\alpha A}} .$$ (2)

As above mentioned, the manner in which the stress in the tooth root changes during the contact period depends on kinematic parameters and tooth geometry, as well as on load distribution.

The analysis of load distribution in simultaneously meshed teeth pairs can be conducted on the basis of the load distribution of the non-dimensional factor. The load distribution factor in simultaneously meshed teeth pairs is defined using the load ratio:

$$K_{\alpha y} = \frac{F_y}{F} \tag{3}$$

The characteristic profile contact points of the simultaneously meshed teeth pairs at which load transfer is carried out, when during the contact period a double and a single mesh follow each other ($1 < \varepsilon_\alpha < 2$), are shown in Figs. 1 and 2.
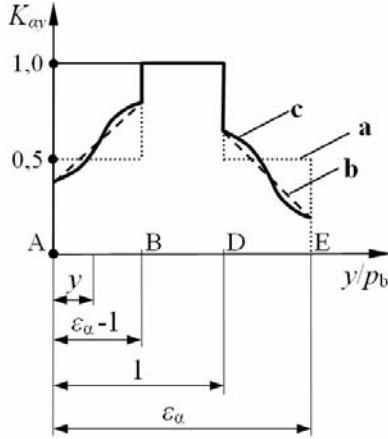

Fig. 2 – Tooth load distribution at the single and double mesh (a – theoretical, b – approximated, c – actual).

The load distribution factor changes when during the contact period a double and a single mesh follow each other, as shown in Fig. 2. The line in Fig. 2 shows the theoretical load distribution, when the teeth are ideally accurate in shape and dimensions, and are absolutely stiff. Load distribution, in real gear teeth pairs, is shown by the line c – real load distribution. The approximation of real load distribution is indicated by the line b.

3. RELATIVE STRESS FACTOR

To view only a partial influence of the tooth geometry on the tooth root stress, a load of equal intensity was observed acting at each tooth profile point. In other words, all the tooth profile points participate equally in transmitting the gear teeth pair load during the contact period (Fig.1):

$$F_A = F_B = ... = F_y = ... = F_E. \tag{4}$$

Based on this condition and Eq. (3), it follows:

$$K_{\alpha A} = K_{\alpha B} = ... = K_{\alpha y} = ... = K_{\alpha E}. \tag{5}$$

On the basis of Eqs. 2 and 5, the stress ratio in the tooth root can be written in the form:

$$\frac{\sigma_{Fy}}{\sigma_{FA}} = \frac{Y_{Fy}Y_{Sy}}{Y_{FA}Y_{SA}} = Y_{\varepsilon y}, \tag{6}$$

so that the load distribution effect on stress in the tooth root is eliminated. Then, according to Eq. 6, the stress ratio for the contact at any y point and for the contact at point A was used to define the relative stress factor, and it was defined by the ratio of corresponding form factors to stress correction.

The relative stress factor achieves the maximum value for the contact at point A (top of the tooth) and according to (6):

$$Y_{\varepsilon max} = Y_{\varepsilon A} = 1, \text{ if } Y_{\varepsilon A} > 1 \text{ then } Y_{\varepsilon A} = 1. \tag{7}$$

According to Eq. (6), the following exact expressions of the relative stress factor for the meshed gear teeth, can be written

$$Y_{\varepsilon y i} = \frac{Y_{Fy i}Y_{Sy i}}{Y_{FA i}Y_{SA i}} \qquad y \in (A,...,E). \tag{8}$$

This factor is specifically determined for both the pinion teeth and the wheel teeth. To determine the relative stress factors for the characteristic contact points on the meshed teeth profiles according to Eq. (8), the knowledge of the $Y_{FA i}$, $Y_{SA i}$, $Y_{Fy i}$ and $Y_{Sy i}$ factor values at the same points is required. The numerical values of the $Y_{FA i}$, $Y_{SA i}$ factor values can be found in the literature in tabular or diagram forms [2]. For the other characteristic contact points (y), due to a number of varying values (teeth number, profile shift coefficient and the contact ratio) that the $Y_{Fy i}$ and $Y_{Sy i}$ factors depend on, their numerical values cannot be represented in tabular and diagram forms, but should be determined by solving the mentioned equations. This procedure of determining the relative stress factor, in accordance with Eq. (8), is quite complex and

justifiable for highly responsible gear teeth pairs, and is required when investigating the operating and critical tooth root condition and optimizing the gear teeth pairs.
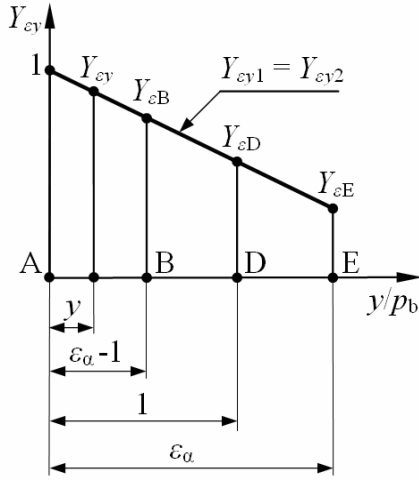

Fig. 3 – Linear change of the relative stress factor during contact period.

To decrease the level of calculation complexity in determining the relative stress factor, the exact model for less responsible gear teeth pairs can be simplified by introducing the following assumptions (Fig. 3):

– relative stress factor equality for the pinion and wheel teeth ($Y_{\varepsilon y1} = Y_{\varepsilon y2}$) and

– linear change of relative stress factor during the contact period.

Based on these assumptions, approximate expressions for the relative stress factor within the characteristic contact points can be created when $1 < \varepsilon_\alpha < 2$:

$$Y_{\varepsilon y} = 1 - \frac{1 - Y_{\varepsilon E}}{\varepsilon_\alpha} \cdot \frac{y}{p_b}. \tag{9}$$

It is well-known that in gear pairs the total load is transmitted through several simultaneously meshed teeth pairs. In spur gear pairs ($\alpha_n = 20°$) at the beginning and the end of the contact period total load is transmitted through two teeth pairs. Around the middle of the contact period total load is transmitted by one teeth pair only. One of the most important steps in calculating tooth root load carrying capacity is the choice of the contact point of simultaneously meshed teeth pairs appropriate for determining tooth root stress. The question is raised: Is it the contact point A or the contact point B, where a single and a double mesh are alternating? The choice of the contact point depends on the load distribution factor in simultaneously meshed teeth pairs and the relative stress factor at points A and B [4,7]. According to Eqs. (3) and (8), the factors $K_{\alpha B} = 1$ and $Y_{\varepsilon A1} = Y_{\varepsilon A2} = 1$. Exact values of the factor $K_{\alpha A}$ can be determined based on the expressions given in [7]. According to Eq. (8), the exact values of the relative stress factor for the contact at point B can be determined using the expressions as follows:

$$Y_{\varepsilon B1} = \frac{Y_{FB1} Y_{SB1}}{Y_{FA1} Y_{SA1}}, \tag{10}$$

$$Y_{\varepsilon B2} = \frac{Y_{FB2} Y_{SB2}}{Y_{FA2} Y_{SA2}}. \tag{11}$$

For the contact at point B ($y = B$) the approximate expression is as follows Eq(9):

$$Y_{\varepsilon B} = 1 - \frac{\varepsilon_\alpha - 1}{\varepsilon_\alpha} \left(1 - Y_{\varepsilon E}\right). \tag{12}$$

In the case $Y_{\varepsilon E} \approx 0.25$, Eq. (12) can be written in the following form:

$$Y_{\varepsilon B} \approx 0.25 + \frac{0.75}{\varepsilon_\alpha}. \tag{13}$$

In the case $Y_{\varepsilon E} \approx 0$, Eq. (12) obtains the form:

$$Y_{\varepsilon B} \approx \frac{1}{\varepsilon_\alpha}. \tag{14}$$

In the literature one can find the approximate Eqs. (13) and (14) and the terms "contact ratio factor" [1, 6], "force arm factor" and "load sharing factor" [12] but not the exact expressions and terms founded upon the basic meaning of these factors. The first term emerged as a consequence of approximate Eqs. (13) and (14) as explicit functions of the contact ratio. The other one is a consequence of the fact that the $Y_{\varepsilon B}$ factor

depends on the teeth bending moment arm value. In the development of gear calculations the approximate Eq. (14) was given firstly in DIN standard in 1970 [17]. Afterwards, the approximate Eq. (13), according to ISO standard, was developed. In the next section, based on the developed exact Eqs. (10) and (11) as well as the approximate Eqs. (13) and (14) for the relative stress factor, the analysis will be carried out on the influence of geometric and kinematic parameters of the gear pair on the tooth root stress.

3.1. Example and results discussion

A comparative analysis of calculation results obtained by the conventional and new mathematical model of calculating the factor Y_ε is realized on a sample of cylindrical gear pair groups I and II. Basic input data of gear pair groups I and II and output data obtained by calculation are shown in Table 1. Calculation was carried out using the MatCad program.

Table 1

Geometric and kinematic parameters of gear pairs

		Gear pairs groups			
Notation	Unit	**Ia**	**Ib**	**IIa**	**IIb**
z_1	-	25	25	25	25
u	-	3	3	6	6
m_n	mm	5	5	5	5
α_n	°	20	20	20	20
h_{a0}	mm	1.25 m	1.25 m	1.25 m	1.25 m
ρ_{a0}	mm	0.25m	0.25m	0.25m	0.25m
b	mm	20	20	20	20
x_1	-	0	0.3	0	0.3
x_2	-	0	-0.3	0	-0.3
ε_α	-	1.790	1.732	1.831	1.755
S_{F1}	mm	10.028	10.753	10.028	10.753
S_{F2}	mm	11.238	10.885	11.594	11.401
ρ_{F1}	mm	2.404	1.890	2.404	1.890
ρ_{F2}	mm	1.933	2.330	1.668	1.928
Y_{SB1}	-	2.073	2.500	2.103	2.532
Y_{SB2}	-	2.435	2.100	2.684	2.367
Y_{FB1}	-	1.358	1.109	1.307	1.078
Y_{FB2}	-	1.160	1.374	1.082	1.246
$(y/p_b)_B$	-	0.790	0.732	0.831	0.755
$(y/p_b)_D$	-	1	1	1	1
$(y/p_b)_E$	-	1.790	1.732	1.831	1.755
$Y_{\varepsilon B1}$	-	0.627	0.639	0.612	0.630
$Y_{\varepsilon B2}$	-	0.662	0.685	0.669	0.695

In the offered new method, each gear in a pair, the pinion and wheel, according to the tooth profile shape, has the appropriate value of the factor $Y_{\varepsilon B}$, which can be determined on the basis of Eqs. (10) and (11). Using the conventional procedure [2] this factor is determined by Eq. (13) for a gear pair. A comparative analysis of the exact values of factor $Y_{\varepsilon y}$ obtained according to Eqs. (10) and (11) and approximate values of this factor obtained by Eqs. (13) and (14) is shown by the diagrams in Figs. 4 and 5. These diagrams show changes of the factor $Y_{\varepsilon y}$ in the contact period using the example of two groups of cylindrical gear pairs. The influence of the teeth profile shape on the factor $Y_{\varepsilon y}$ and the way it changes during the contact period by varying the number of teeth and profile shift coefficient is analyzed. When $u > 1$, and $x_1 = x_2 = 0$, then $Y_{\varepsilon y1} \neq Y_{\varepsilon y2}$, i.e., the tooth root stress in the pinion and wheel does not have the same function of change (Figs. 4a and 5a). These are curves with a different sign of the curvature, intersecting at point P. Consequently, at point P the relative stress factors in the pinion and wheel tooth roots are the same. In the tooth profile interval, between points A and P, the relative stress factor in the wheel tooth root is greater than that in the pinion tooth root, while the gradient of the relative stress factor change is greater in the pinion tooth root than that in the wheel tooth root. In the point P – point E interval, the relative stress factor is greater in the pinion tooth root, whereas the stress gradient change is greater in the wheel tooth root. When $u > 1$, and $x_1 > 0$ and $x_2 < 0$ (Figs. 4b and 5b) point P shifts to point E in the tooth root. In that case, the difference between factors $Y_{\varepsilon y1}$ and $Y_{\varepsilon y2}$ is increased in the point A to point P interval, and after point P it is decreased. Simultaneously, as the gear ratio is increased, the difference between the relative stress factors in the wheel and pinion tooth root is increased. The relative stress factor of the wheel can be 4…10% larger than the relative stress factor

of the pinion, depending on the teeth number and profile shift coefficient (Table 2). Approximate values of the relative stress factor, determined according to Eqs. (13) and (14), are indicated by dashed and dotted lines, respectively, in Figs. 4 and 5. Table 2 gives a comparative analysis of the values of factor $Y_{\varepsilon exactB}$, obtained by the developed mathematical model and the model obtained using the conventional method. According to this analysis, the difference in the values of factor $Y_{\varepsilon exactB}$ and "contact ratio factor" Y_ε ISO in the B point vary between 3.9% and 7.5% for the pinion and 0.3% and 3.9% for the wheel.

Table 2

Comparative values of factor Y_B
according to the developed mathematical model and ISO

Gear pairs groups				
Notation	**Ia**	**Ib**	**IIa**	**IIb**
z_1	25	25	25	25
U	3	3	6	6
x_1	0	0.3	0	0.3
x_2	0	-0.3	0	-0.3
Y_ε ISO (B point)	0.669	0.683	0.660	0.677
$Y_{\varepsilon\, exactB1}$	0.627	0.639	0.612	0.630
Deviation in % (pinion)	6.7	6.9	7.8	7.5
$Y_{\varepsilon\, exactB2}$	0.662	0.685	0.669	0.695
Deviation in % (wheel)	1.1	0.3	1.3	2.6
$Y_{\varepsilon\, exactB2}/Y_{\varepsilon\, exactB1}$	1.06	1.07	1.09	1.10

Based on the performed analysis, it has been shown that Eq. (13) for factor Y_ε corresponds more closely to the real state than Eq. (14) applied for the first time in the conventional method for calculating teeth load [17].
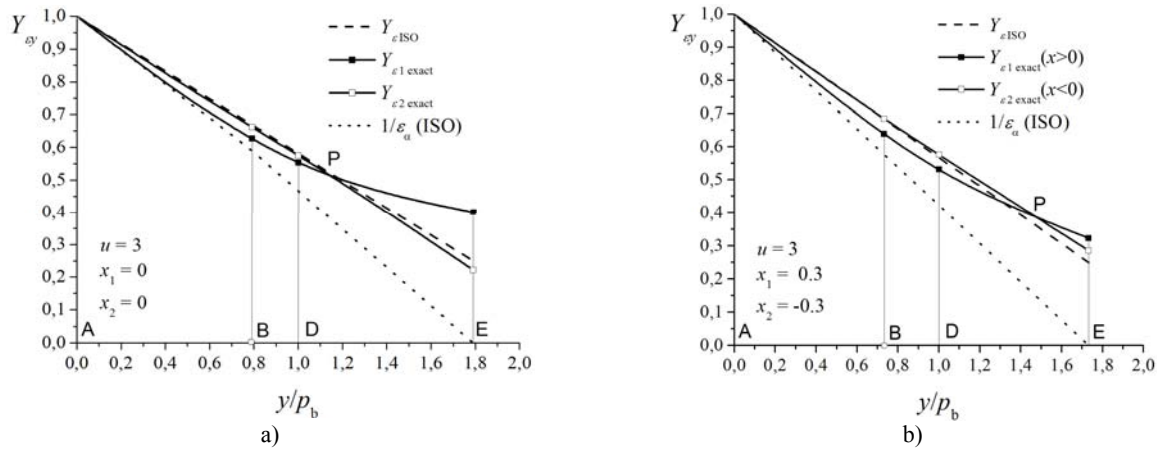
Fig. 4 – Change of $Y_{\varepsilon y}$ factor during the contact period ($u = 3$; $x_1 = 0$, 0.3; $x_2 = 0$, – 0.3).
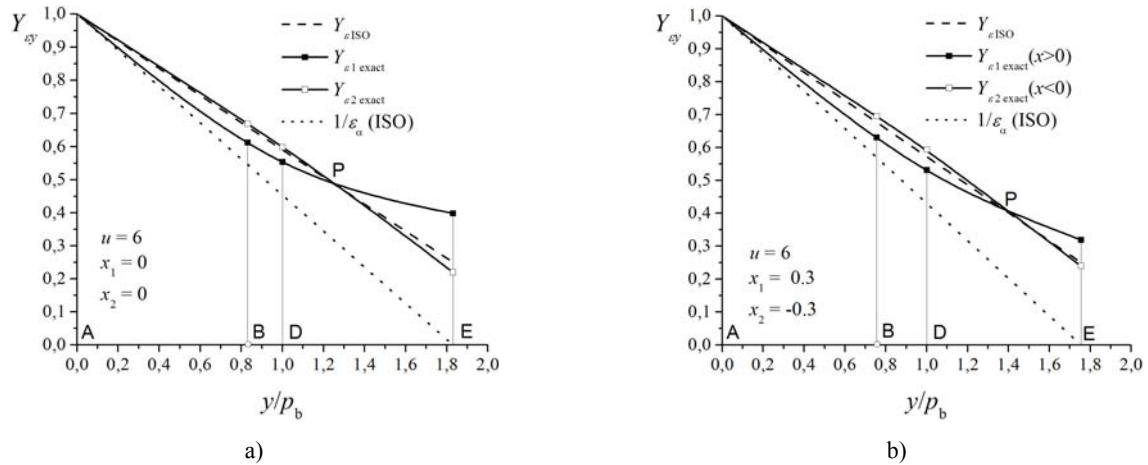
Fig. 5 – Change of $Y_{\varepsilon y}$ factor during the contact period ($u = 6$; $x_1 = 0$, 0.3; $x_2 = 0$, – 0.3).

A comparative analysis of the exact values of factor $Y_{\varepsilon B}$, obtained according to Eq. (10) and (11) and approximate values of this factor obtained by Eqs. (13) and (14), is shown by diagrams in Figs. 6 and 7, depending on the contact ratio ε_α, profile shift coefficient and teeth number.
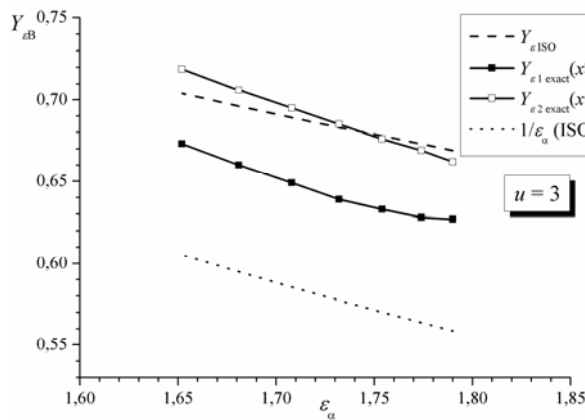

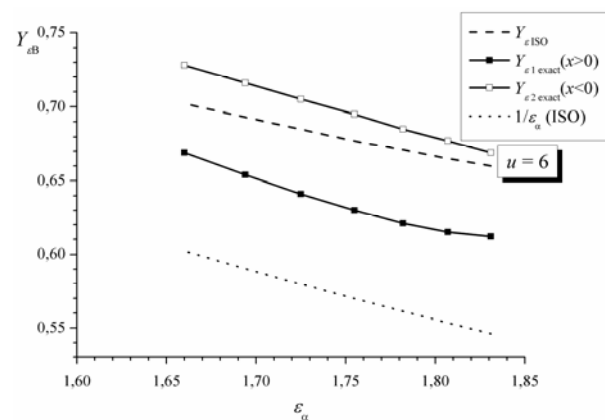Fig. 6 – Factor $Y_{\varepsilon B}$ *vs.* contact ratio ε_α ($u = 3$). 　　　Fig. 7 – Factor $Y_{\varepsilon B}$ *vs.* contact ratio ε_α ($u = 6$).

The difference between the approximate and exact values of factor Y_ε in the B point for the wheel is much lower than for the pinion, as it is shown by diagrams in Figs. 6 and 7. When gear ratio is greater than one, the difference of the relative stress factor between the wheel ($Y_{\varepsilon2exact}$) and pinion ($Y_{\varepsilon1exact}$) does not depend on the transverse contact ratio. Also this difference increases as the gear ratio increases.

4. CONCLUSION

On the basis of investigation results obtained in this work, it has been shown that geometric and kinematic parameters of the spur gear pairs can significantly affect the tooth root stress. To this end, the original analytical model is developed. In the analytical model, the relative stress factor is defined. Simple expressions for exact and approximate determination of the relative stress factor at each point of the tooth profile are derived. The analysis of this factor is used to show that kinematic and geometric parameters have stronger influence on the tooth root stress of the pinion than on that of the wheel, when the contact points are located on the addendum (the area of large stresses in the tooth root). When the contact points are located on the dedendum (the area of small stresses in the tooth root), kinematic and geometric parameters have stronger influence on the tooth root stress of the wheel. The greatest difference of the relative stress factor between the pinion and wheel is generated at point B, where a double mesh is alternating with a single mesh. It is well-known that kinematic and geometric parameters at point B are significant for determining the stress authoritative in the control of tooth load carrying capacity. When gear ratio is greater than one, the difference of the relative stress factor between the pinion and wheel does not depend on the transverse contact ratio. This difference increases as the gear ratio increases.

In this paper, it is shown that the values of the "contact ratio factor" Y_ε, determined based on the approximate expression according to ISO standard, deviate compared to exact values obtained using the defined relative stress factor. These deviations fall within the 0.3–7.5% interval. In that case, the deviations are greater in the pinion gear teeth than in the wheel gear teeth. Also, it is shown that the approximate expression for the "contact ratio factor" in the ISO procedure was derived assuming that it changes linearly during the contact period and that its values are approximately identical for the pinion and wheel gear teeth. Substituting the "contact ratio factor" by the relative stress factor would increase the accuracy of stress calculation for the tooth root according to standard procedure, and physicality of the contact ratio factor would be explained. Also, the results of carried out investigation can be implemented in the convetional B method of the ISO calculation procedure. The developed analytical model can be used for the choice of optimal kinematic and geometric parameters of the gear pair in terms of the tooth root stress, as well as for optimisation of power transmissions [18].

ACKNOWLEDGMENTS

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