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DEFERRED STATISTICAL CONVERGENCE ON TIME SCALES

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Abstract. In this study, we introduce deferred statistical convergence and strongly deferred Cesàro summability on an arbitrary time scale. Moreover, we examine some relationship between deferred statistical convergence and strongly deferred Cesàro summability on time scales.

Key words: statistical convergence, time scales, Cesàro summability.

1. INTRODUCTION

Convergence is one of the most important concepts in mathematics. It is of strong theoretical importance in many fields. In applied fields, this concept is indispensable. For this reason, different types of convergence have been introduced over the years and important results and concepts have emerged. One of these concepts is statistical convergence. The thought of statistical convergence was given by Zygmund [29] in 1935. Statistical convergence was introduced by Steinhaus [24] and Fast [11] and reintroduced by Schoenberg [21] independently. In following years and under diverse names, it has been discussed Fourier analysis, Ergodic, Number, Turnpike and Measure theories, Trigonometric series, Banach spaces. In the next process, it was further investigated from the sequence space point of view and linked with summability theory by Altın *et al.* [2], Cinar *et al.* ([7], [10]), Connor [8], Denjoy [9], Fridy [12], Işık and Akbaş [3, 15], Küçükaslan and Yılmaztürk [17], Nuray [18], Salat [20] and many others. The relationship between statistical convergence and classical summability continued to be studied for many years. First, let's remind statistical convergence in the classical sense.

Statistical convergence depends upon density of subsets of $\mathbb{N}$. The density of $\mathbb{E} \subset \mathbb{N}$ is defined by

$$\delta(\mathbb{E}) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_{\mathbb{E}}(k),$$

provided that limit exists. Here, $\chi_{\mathbb{E}}$ is characteristic function of $\mathbb{E}$. It is obvious that any finite subset of $\mathbb{N}$ has zero natural density and

$$\delta(\mathbb{E}^c) = 1 - \delta(\mathbb{E}).$$

$x = (x_k)_{k\in\mathbb{N}}$ is said to be statistically convergent to L if

$$\delta(\{k \in \mathbb{N} : |x_k - L| \geqq \varepsilon\}) = 0,$$

for each $\varepsilon > 0$. One can write $x_k \xrightarrow{\text{stat}} L$ as $k \to \infty$ or $S - \lim_{k\to\infty} x_k = L$. This definition has been expressed in different ways over the years and its relationship with aggregation has been examined in some areas. In recent years, relationship between statistical convergence and summability theory has been tried to be carried into applied fields. Now, it is time to remember deferred Cesàro mean and deferred statistical convergence, which are the basic concepts of our study.

Agnew [1] introduced deferred Cesàro mean of real or complex valued sequences $x=(x_k)$ in 1932 as

$$(D_{p,q}x)_n = \frac{1}{(q_n - p_n)} \sum_{k=p_n+1}^{q_n} x_k, \, n = 1,2,3,\ldots,$$

where $p=(p_n)$ and $q=(q_n)$ are the sequences of non-negative integers satisfying

$$p_n < q_n \quad \text{and} \quad \lim_{n\to\infty} q_n = \infty. \tag{1}$$

Let $K \subset \mathbb{N}$ and $K_{p,q}(n) = \{k : p_n < k \le q_n, k \in K\}$. Deferred density of K is defined by

$$\delta_{p,q}(K) = \lim_{n\to\infty} \frac{1}{(q_n - p_n)} \left| K_{p,q}(n) \right|,$$

when the right side limit exists. The vertical bars indicate the cardinality of $K_{p,q}(n)$. If $q_n = n$, $p_n = 0$, deferred density coincides with natural density of K.

$x=(x_k)$ is said to be deferred statistically convergent to L, if

$$\lim_{n\to\infty} \frac{1}{(q_n - p_n)} \left| \{p_n < k \le q_n : |x_k - L| \ge \varepsilon\} \right| = 0,$$

for each $\varepsilon > 0$ [16]. Here, one can write $S_{p,q} - \lim x_k = L$. If $q_n = n$, $p_n = 0$, deferred statistical convergence coincides with usual statistical convergence [17]. This type of convergence includes many convergences at the same time and it is possible to switch to other convergences depending on the change of p and q. Here, the intersection process of time scale and summability theories and the work done in this field will be briefly mentioned.

The time scale theory was given by Hilger in 1988 [14]. The details of the time scale theory were first given in detail by [5] in 2001. This study accelerated the studies on time scale theory. Later, Guseinov [13] was constructed measure theory on time scales in 2003. Lebesque $\Delta-$integral introduced by Cabada and Vivero [6] in 2006 on time scales. These studies are the basis for the studies on summability theory on time scales. Over the years, the time scale calculus has been studied by many mathematicians in different fields [4]. So, in view of recent applications of time scales for real life problems, it seems usual to generalize convergence on time scales. Statistical convergence is applied to time scales for different aims by numerous authors in literature (see [22,23,25–28]). Here, our purpose is to move some concepts and properties on deferred statistical convergence to time scale theory. Before moving on to the main topic, let's remember some important concepts related to time scale theory.

A time scale $\mathbb{T}$ is an arbitrary, nonempty, closed subset of real numbers. For $t \in \mathbb{T}$, forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$ and $\rho(t) = \sup\{s \in \mathbb{T} : s < t\}$, respectively. A closed interval on $\mathbb{T}$ is defined $[a,b]_{\mathbb{T}} = \{t \in \mathbb{T} : a \le t \le b\}$. Now, let B denote the family of $[a,b)_{\mathbb{T}} \in \mathbb{T}$ and $m : B \to [0,\infty)$ be the set function on B where $m([a,b)_{\mathbb{T}}) = b - a$. Then, it is known that m is a countably additive measure on B. Caratheodory extension of m associated with B is said to be Lebesque $\Delta-$measure on $\mathbb{T}$ and is denoted by μ_Δ (see [6], [19]). The properties of $\Delta-$measure [13] are as follows;

i) If $a \in \mathbb{T} \backslash \max\{\mathbb{T}\}$, then single point set $\{a\}$ is $\Delta-$measurable and its $\Delta-$measure is $\mu_\Delta(a) = \sigma(a) - a$,

ii) If $a,b \in \mathbb{T}$ and $a \le b$, then $\mu_\Delta([a,b)_{\mathbb{T}}) = b - a$ and $\mu_\Delta((a,b)_{\mathbb{T}}) = b - \sigma(a)$,

iii) If $a,b \in \mathbb{T} \backslash \max\{\mathbb{T}\}$ and $a \le b$, then $\mu_\Delta((a,b]_{\mathbb{T}}) = \sigma(b) - \sigma(a)$ and $\mu_\Delta([a,b]_{\mathbb{T}}) = \sigma(b) - a$.

These important features will be used effectively throughout the study. These definitions and properties are of vital importance for the study of statistical convergence on time scales and its relation to summability in this field. $\Delta-$measurement and all its features are extremely important in terms of the accuracy of the results to be obtained.

2. DEFERRED STATISTICAL CONVERGENCE ON TIME SCALES

In this section, deferred statistical convergence and strongly deferred Cesàro summability are defined and their basic principles are established on an arbitrary time scale. In the following theorems and definitions, (p_n) and (q_n) will be assumed to be sequences of non-negative integers satisfying (1) up to Theorem 5.

Definition 1. Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable. f is deferred statistically convergent on $\mathbb{T}$ to a number L if

$$\lim_{n\to\infty}\frac{\mu_\Delta\{s\in(p_n,q_n]_\mathbb{T}:|f(s)-L|\geqslant\varepsilon\}}{\mu_\Delta\left((p_n,q_n]_\mathbb{T}\right)}=0,$$

for each $\varepsilon>0$. We can write $DS^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}f(t)=L$.

Throughout this study, set of all deferred statistically convergent functions on $\mathbb{T}$ will be indicated by $DS^{\mathbb{T}}_{[p,q]}$. Additionally, it is clear that:

i) If $q(t)=t$, $p(t)=t_0$ then we get statistical convergence on $\mathbb{T}$ [25],

ii) If $q_n=k_n$, $p_n=k_{n-1}$, where (k_n) is a lacunary sequence, then we get lacunary statistical convergence on $\mathbb{T}$ [26],

iii) If $q_t=t$, $p_t=t-\lambda_t+t_0$, then we get $\lambda-$statistical convergence on $\mathbb{T}$ [27].

These situations clearly show the importance of our study. Significant types of statistical convergence in the current classical situation are thus generalized using the time scale.

THEOREM 1. *Let us consider $f,g:\mathbb{T}\to\mathbb{R}$ with $DS^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}f(t)=L_1$ and $DS^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}g(t)=L_2$. Then, the below statements hold:*

i) $DS^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}\big(f(t)+g(t)\big)=L_1+L_2$,

ii) $DS^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}\big(cf(t)\big)=cL_1, c\in\mathbb{R}$.

Proof. Omitted. □

Definition 2. Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable. Then, f is strongly deferred Cesàro summable to L if

$$\lim_{n\to\infty}\frac{1}{\mu_\Delta\big((p_n,q_n]_\mathbb{T}\big)}\int_{(p_n,q_n]_\mathbb{T}}|f(s)-L|\Delta s=0.$$

Here, we write $DW^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}f(t)=L$. $DW^{\mathbb{T}}_{[p,q]}$ denotes set of all deferred Cesàro summable functions on $\mathbb{T}$.

THEOREM 2. *Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable. If $f\in DW^{\mathbb{T}}_{[p,q]}$, then $f\in DS^{\mathbb{T}}_{[p,q]}$.*

Proof. Suppose that $DW^{\mathbb{T}}_{[p,q]}-\lim\limits_{t\to\infty}f(t)=L$. For every $\varepsilon>0$, it yields that

$$\begin{aligned}\int_{(p_n,q_n]_\mathbb{T}}|f(s)-L|\Delta s&\geqslant\int_{\{s\in(p_n,q_n]_\mathbb{T}:|f(s)-L|\geqslant\varepsilon\}}|f(s)-L|\Delta s\\&\geqslant\varepsilon\ \mu_\Delta\{\{s\in(p_n,q_n]_\mathbb{T}:|f(s)-L|\geqslant\varepsilon\}\}.\end{aligned}$$

□

THEOREM 3. *Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable. If $f\in DS^{\mathbb{T}}_{[p,q]}$ and f is bounded, then $f\in DW^{\mathbb{T}}_{[p,q]}$.*

Proof. Assume that f is bounded and deferred statistical convergent to L on $\mathbb{T}$. Then, there exists a number $K>0$ such that $|f(s)-L|\leqslant K$ and

$$\lim_{n\to\infty}\frac{1}{\mu_\Delta\big(p_n,q_n]\big)}\mu_\Delta\big(\{s\in(p_n,q_n]:|f(s)-L|\geqslant\varepsilon\}\big)=0.$$

Therefore, we have

$$\begin{aligned}\frac{1}{\mu_\Delta((p_n,q_n])}\int_{(p_n,q_n]_{\mathbb{T}}}|f(s)-L|\Delta s &= \frac{1}{\mu_\Delta((p_n,q_n])}\int_{\{(p_n,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}}|f(s)-L|\Delta s\ + \\ &+\frac{1}{\mu_\Delta((p_n,q_n])}\int_{\{(p_n,q_n]_{\mathbb{T}}:|f(s)-L|<\varepsilon\}}|f(s)-L|\Delta s\ \leqslant \\ &\leqslant \frac{K}{\mu_\Delta((p_n,q_n])}\int_{(p_n,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}}\Delta s+\frac{\varepsilon}{\mu_\Delta((p_n,q_n])}\int_{[p_n,q_n]_{\mathbb{T}}}\Delta s\ = \\ &= \frac{K\ \mu_\Delta\big(\{s\in(p_n,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}\big)}{\mu_\Delta\big((p_n,q_n]\big)}+\varepsilon.\end{aligned}$$

It completes the proof as $n\to\infty$. □

THEOREM 4. *Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable, and $\frac{\mu_\Delta\big((p_n,q_n]\big)}{\sigma(q_n)}$ be bounded. If f is statistical convergent to L on $\mathbb{T}$, then f is deferred statistical convergent to L on $\mathbb{T}$.*

Proof. Since f is statistical convergent to L on $\mathbb{T}$, we can write

$$\begin{aligned}\frac{1}{\mu_\Delta\big((t_0,q_n]\big)}\mu_\Delta\big(\{s\in(t_0,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}\big) &\geqslant \frac{\mu_\Delta\big(\{s\in(p_n,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}\big)}{\sigma(q_n)-t_0} \\ &\geqslant \frac{\sigma(q_n)-\sigma(p_n)}{\sigma(q_n)}\frac{\mu_\Delta\big(\{s\in(p_n,q_n]_{\mathbb{T}}:|f(s)-L|\geqslant\varepsilon\}\big)}{\sigma(q_n)-\sigma(p_n)}\end{aligned}$$

If we take limit as $n\to\infty$, then the theorem is proved. □

COROLLARY 1. *Let $f:\mathbb{T}\to\mathbb{R}$ be Δ-measurable, (q_n) be an arbitrary sequence with $q_n\in[t_0,t]$ and $\frac{\mu_\Delta\big([t_0,t]\big)}{\mu_\Delta\left((p_n,q_n]\right)}$ be bounded. If f is statistical convergent to L on $\mathbb{T}$, then f is deferred statistical convergent to L on $\mathbb{T}$.*

THEOREM 5. *Let $p_n,q_n,p_n^{'}$ and $q_n^{'}$ be sequences of non-negative integers satisfying*

$$p_n\leq p_n^{'}<q_n^{'}\leq q_n \tag{2}$$

for all $n\in\mathbb{N}$ such that

$$\lim_{n\to\infty}\frac{\mu_\Delta\big((p_n,q_n]\big)}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}=a>0. \tag{3}$$

Then, $f\in DS^{\mathbb{T}}_{[p,q]}$ implies $f\in DS^{\mathbb{T}}_{[p',q']}$.

Proof. Assume that $DS^{\mathbb{T}}_{[p,q]}-\lim_{t\to\infty}f(t)=L$. Since (2) is provided, we have

$$\{s\in(p_n^{'},q_n^{'}]:|f(s)-L|\geqslant\varepsilon\}\subset\{s\in(p_n,q_n]:|f(s)-L|\geqslant\varepsilon\},$$

for given $\varepsilon>0$ and also the following inequality:

$$\begin{aligned}\frac{1}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}\mu_\Delta\big(\{s\in(p_n^{'},q_n^{'}]:|f(s)-L|\geqslant\varepsilon\}\big) &\leqslant \frac{1}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}\mu_\Delta\big(\{s\in(p_n,q_n]:|f(s)-L|\geqslant\varepsilon\}\big)\leqslant \\ &\leqslant\frac{\mu_\Delta\big((p_n,q_n]\big)}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}\frac{1}{\mu_\Delta\big((p_n,q_n]\big)}\mu_\Delta\big(\{s\in(p_n,q_n]:|f(s)-L|\geqslant\varepsilon\}\big)\end{aligned}$$

Taking limit as $n\to\infty$ and using (3), we get $DS^{\mathbb{T}}_{[p',q']}-\lim\limits_{t\to\infty} f(t)=L$. □

THEOREM 6. *Let $p_n, q_n, p_n^{'}$ and $q_n^{'}$ be sequences of non-negative integers satisfying* (2) *such that*

$$\lim_{n\to\infty}\frac{\mu_\Delta\big((p_n,p_n^{'}]\big)}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}=0,\ \lim_{n\to\infty}\frac{\mu_\Delta\big((q_n^{'},q_n]\big)}{\mu_\Delta\big((p_n^{'},q_n^{'}]\big)}=0. \tag{4}$$

If f is bounded, $f\in DS^{\mathbb{T}}_{[p',q']}$ implies $f\in DW^{\mathbb{T}}_{[p,q]}$.

Proof. Suppose that $DS^{\mathbb{T}}_{[p',q']}-\lim\limits_{t\to\infty} f(t)=L$ and f is bounded. Since f is bounded, there exists a number $M>0$ such that $|f(s)-L|\leqslant M$. Then, we may write

$$\begin{aligned}
&\frac{1}{\mu_\Delta((p_n,q_n])}\int\limits_{[p_n,q_n]_{\mathbb{T}}}|f(s)-L|\Delta s\\
&=\frac{1}{\mu_\Delta((p_n,q_n])}\Big[\int\limits_{[p_n,p_n^{'}]_{\mathbb{T}}}|f(s)-L|\Delta s+\int\limits_{[p_n^{'},q_n^{'}]_{\mathbb{T}}}|f(s)-L|\Delta s+\int\limits_{[q_n^{'},q_n]_{\mathbb{T}}}|f(s)-L|\Delta s\Big]\\
&\leqslant\frac{1}{\mu_\Delta((p_n^{'},q_n^{'}])}\Big[\int\limits_{[p_n,p_n^{'}]_{\mathbb{T}}}|f(s)-L|\Delta s+\int\limits_{[p_n^{'},q_n^{'}]_{\mathbb{T}}}|f(s)-L|\Delta s+\int\limits_{[q_n^{'},q_n]_{\mathbb{T}}}|f(s)-L|\Delta s\Big]\\
&\leqslant\frac{M}{\mu_\Delta((p_n^{'},q_n^{'}])}(\sigma(p_n^{'})-\sigma(p_n)+(\sigma(q_n)-\sigma(q_n^{'})\\
&+\frac{1}{\mu_\Delta((p_n^{'},q_n^{'}])}\Big[\int\limits_{\{[p_n,q_n]:|f(s)-L|\geqslant\varepsilon\}}|f(s)-L|\Delta s+\int\limits_{\{[p_n,q_n]:|f(s)-L|<\varepsilon\}}|f(s)-L|\Delta s\Big]\\
&\leqslant\frac{\mu_\Delta((p_n,p_n^{\prime}])+\mu_\Delta((q_n^{\prime},q_n])}{\mu_\Delta((p_n^{'},q_n^{'}])}M\\
&+\frac{M}{\mu_\Delta((p_n^{'},q_n^{'}])}\mu_\Delta(\{p_n^{\prime}<k\leqslant q_n^{\prime}:|f(s)-L|\geqslant\varepsilon\})+\frac{\varepsilon}{\mu_\Delta((p_n^{'},q_n^{'}])}
\end{aligned}$$

Taking limit as $n\to\infty$ and using (4), we get $DW^{\mathbb{T}}_{[p',q']}-\lim\limits_{t\to\infty} f(t)=L$. □

3. CONCLUSION

The basic thought of statistical convergence of a sequence is that majority of its terms converges and we do not care what is going on with other terms. Statistical convergence is directly related to convergence of such statistical characteristics as mean and standard deviation. Therewithal, it is known that sequences that come from measurement and computation, do not allow, in a general case, to test whether they converge or statistically converge in the strict mathematical sense. Over the years, different versions of statistical convergence have been examined and very important results have been obtained. One of these versions is deferred statistical convergence. In this study, this variant of statistical convergence is examined on arbitrary time scales and an important generalization is made. The present results thus become a special case of our results. Then, strongly deferred Cesàro summability is constructed on time scales. Finally, some inclusion relations are studied for the new obtained spaces. As the time scale differs, the definitions and theorems discussed will change. In many ways, this will make a difference to applications involving the theory of summability.

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