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QUANTUM ADAPTIVE APPROACH TO CHORD CONSONANCE MODELING

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Abstract. In this paper, a chord consonance model is established via complex law analysis to improve the accuracy of music emotion calculation. To address the coupling and the uncertainty of the complex law, quantum adaptive method is applied to the chord consonance calculation. Musical tones, with subjective difference and uncertainty, are constructed into interval consonance quantum state using parameter adaptive method. Quantum logic gates are introduced to describe the coupling between musical tones to establish the chord consonance model. Then, a mixed state density matrix is designed to optimize the calculation of consonance. Experimental results are provided to verify the validity and accuracy of the modelling.

Key words*:* quantum logic gates, adaptive parameters, chord consonance.

1. INTRODUCTION

The degree of chord consonance describes the subjective perception of chords on hearing; the higher the degree is, the more stable and pleasant the sense of hearing becomes. The opposition between consonance and inconsonance denotes the foundation of western music creation [1]. Based on the complexity of the musical system, the non-linearity of pitch frequency perception, the uncertainty of subjective differences, and the coupling between intervals are considered to accurately divide the degree of the interval and the chord consonance. Musical information such as interval frequency ratio and sound group can be applied to quantitatively describe the degree of chord consonance, which improves the tension and hierarchy of musical works, meanwhile, reduces the ambiguity of traditional music emotion calculation [2]. Many studies have been conducted concerning the degree of interval consonance [3]. However, due to the coupling between intervals, the degree of chord consonance is difficult to quantify, leading to the less involvement of the research on chord consonance. The chord consonance description is usually proposed based on musicology, psychology, brain science, and physical calculation due to its interdisciplinary nature. A purely quantitative modeling method for the degree of chord consonance is uncommon due to the coupling between the intervals. Furthermore, due to the complexity and importance of decision-making, data manipulation based on modeling and algorithms can aid in the resolution of practical problems. Therefore, the goal of this paper is to develop a purely quantitative method for describing chord consonance using modeling and an algorithm.

Zhang-Cai Long et al. used laser interferometry to conduct various studies on the physiological process of auditory perception, and proved that the virtual perceived pitch and actual pitch are generally consistent but not completely linear considering individual differences [6]. Due to the non-linearity in the perception of pitch frequencies in different pitch intervals, the calculation complexity of the degree of harmony is increased [7]. Subjective differences in auditory perception lead to parameter uncertainty in the interval consonance model. Peter M.C. Harrison et al. combined cultural familiarity, overtone interference and periodic/harmonic characteristics via non-standardized regression coefficients, then proposed a composite chord consonance model based on the given corpus and multiple influencing factors to comprehensively improve the designation accuracy of the model in specific musical context [8]. However, due to the simplification of the corpus, the accuracy of the model still needs improvement in case of the alternation of the music context [6]. To solve the problem of the consonance degree of pure-tone interval, Trulla et al. used recursive quantitative analysis to identify the time delay of the repetitive period of the waveform, and replaced the traditional frequency ratio for the quantitative analysis of interval consonance degree [9]. The new ideas are introduced for the detection of consonance, but the effect of overtones had not been considered [10]. Because the traditional frequency ratio model [11–13] cannot describe the degree of interval consonance with harmonic wave, hence, the accuracy of the model is reduced. It is difficult to choose a suitable mathematical model to describe due to the coupling among intervals in the chords. According to the construction of the music model, the probabilistic characteristics of the quantum model makes it more tolerable and less complex [14]. Aiming at the problem of tonal attraction, Reinhard Blutner et al. overcame the methodological and empirical short-comings of the classical ICP model by integrating quantum probability theory and group theory based on cognitive state projection properties, then proposed an optimized probability model based on quantum cognition [15], which provides research ideas for the description of the coupling characteristics between intervals in chords in this paper. Many optimization problems exhibit complex properties such as multi-modality and high linearity in most cases, and many researchers are working on intelligent and self-adaptive optimization strategies. Therefore, it is critical to select appropriate control parameters [16], which provide a solution to the non-linear problem of audio perception. Numerous studies on the description of chord consonance have been conducted, the majority of which were based on the calculation of single and pure tones, with complex tones with harmonics frequently being ignored. Furthermore, the non-linearity of pitch perception and subjective listening uncertainty are left unaddressed. However, the description of chord consonance calculation is inadequate, and research on the seventh and ninth chords is lacking, as is consideration of the effect of interval coupling on chord consonance. The ultimate goal of this paper is to create a chord consonance modeling method that takes interval coupling and harmonic factors into account. In this study, the calculation parameters of chord consonance were automatically adjusted according to the traditional chord consonance characteristics and in combination with a quantum adaptive algorithm that allowed for high calculation tolerance, based on the quantum model proposed by previous researchers (i.e., low calculation complexity). Therefore, this method can be incorporated into the statistical distribution and structural characteristics of chord consonance to overcome the non-linearity of pitch perception and the uncertainty of subjective listening, resulting in the development of a highly accurate chord consonance model that takes into account the coupling between intervals and harmonics.

Considering the nonlinearity of pitch perception, the uncertainty of subjective hearing and the coupling between intervals, the calculation scheme of chord consonance based on quantum adaptive method is studied [17]; This paper proposes three models that describe pure tone interval consonance, complex tone interval consonance, and chord consonance, respectively, and the chord consonance model is the result of these three models combined. The modeling steps are given in Fig. 1 [30]. First, an iterative algorithm is proposed to fit the pure tone harmonic model using the simplest frequency ratio. Second, the harmonic wave is considered based on the pure tone harmonic model, and by designing rules for selecting the harmonic pairs, the pure tone consonance model is used to calculate multiple harmonics, and the superposition of fundamental and higher harmonics is considered to introduce the quantum state probability characteristics to optimize the model in order to construct a complex interval consonance model that not only considers the harmonic factors but also is based on the quantum state probability characteristics [18]. In comparison to previous models, this one addresses the issue that the harmonic factor in pitch consonance was frequently overlooked. Therefore, it solves the pitch calculation coupling problem where there are fundamental and higher harmonic superpositions, improves the accuracy of the interval consonance calculation concerning the uncertainty of the hearing difference. Finally, the quantum revolving door is introduced to create a chord consonance model based on the complex tone interval consonance model, where the chord is regarded as a quantum state to be transformed in the chord system with intervals as the initial state [19]. The ultimate goal of this paper is to propose this model. Its contribution is in the quantitative description of the coupling between two intervals in a chord, which avoids the description ambiguity caused by traditional models' inability to accurately quantify [2]. Because most interval harmony research has concentrated on pure tones, the main contribution of this paper is the chord harmony model developed by constructing a complex tone interval harmony model. Mustafa Gerger et al. achieved a Spiral Test Based on Pattern Recognition accuracy of 1.00 in diagnosing

Parkinson's disease, indicating that artificial intelligence has been highly successful in many fields in assisting researchers in making decisions in complex situations [20]. With the advancement of automatic music retrieval, automatic music recognition and generation, music semiotics, and music visualization, it is imperative that definitive musical quantitative analysis be extended to the field of music. The higher precision chord consonance model proposed in this paper has a wide range of applications in music artificial intelligence and music information. This study’s technical contributions are summarized below:

1. The adaptive method was used to fine-tune the model parameters in order to improve the accuracy of the interval consonance calculation.

2. To solve the coupling problem of interval calculation with the fundamental wave and higher harmonics superpositions, a complex interval consonance model was built based on the probability characteristics of the quantum states.

3. The chord consonance model was built on the quantum revolving door based on the complex tone interval consonance model proposed in this paper, which solved the problem of being unable to quantitatively describe two intervals in a chord due to coupling.

The modeling process of pure tone interval and complex tone interval consonance is described in the second section of this paper. The third section shows how to model chord consonance based on interval consonance development. The fourth section introduces the verification and testing of interval consonance, the parameter tuning process, and the convergence, which describes the accuracy of the interval model. The verification and testing of chords, as well as the benefits of the model proposed in this paper, are explained. The fifth section summarizes the paper and emphasizes its scientific significance.

2. INTERVAL CONSONANCE MODELING

Interval consonance includes pure-tone interval consonance and compound interval consonance, in which compound tone is composed of overtone and pure-tone. The twelve-tone equal temperament defines the frequency difference corresponding to the pure-tone interval, and the pure-tone interval consonance cannot be directly obtained from the frequency difference [21]; The puretone consonance of different sound groups has non-linear pitch frequency perception which makes the interval frequency difference unable to be unified [22], so the interval consonance of different sound groups cannot be calculated through the interval consonance model for single sound group; Considering the subjective differences of auditory perception, the linear gain parameter uncertainty occurs to the interval consonance model; Due to the harmonic interference of the overtone in the composite tone, the calculation for the consonance of the composite tone interval cannot be calculated accurately.

Aiming at the problem that the pure-tone interval consonance cannot be directly obtained from the frequency difference, the relationship between the pure-tone interval consonance and the interval frequency difference in the same sound group is calculated via the iterative algorithm and the least square method; Considering the problem of non-linear pitch frequency perception which leads to the interval frequency difference in each tone group, the pure-tone interval consonance of different tone groups is calculated via interval frequency difference normalization. For the uncertainty of subjective perception differences, the parameter adaptive method is adopted to improve the accuracy of the calculation of the interval consonance model; In the light of the harmonic interference of the overtone in the composite tone, the calculation model of the interval consonance degree is optimized by calculating the consonance degree of the harmonic pair and introducing the mixed state density matrix. The calculation process of interval consonance in different tone groups is shown in Fig. 2 [30].

2.1. Pure-tone interval consonance degree calculation

The consonance of overtone harmonic pairs in a compound tone can be obtained via pure-tone interval consonance model, which is also the premise of establishing a chord consonance model.

Considering that there is no quantitative description for the “Simplest frequency ratio” theory, the pure-tone interval in a single tone group is taken as the research object, iterative method and function fitting method are used to construct a pure-tone interval consonance model in a single tone group. This paper calculates the interval consonance in a single sound group by calculating the simplest frequency ratio, dividing the degree of consonance and fitting the expression of consonance.

(1) *Simplest frequency ratio calculation*. According to the “Simplest frequency ratio” theory, the closer the ratio of the two tones is to the integer ratio, the more harmonious the interval formed by the two tones becomes; Due to the lack of specific calculation method to obtain the integer ratio of interval frequency, an iterative algorithm is designed to obtain the simplest interval frequency ratio in order to divide the degree of interval consonance.

The threshold $m\%$ is used to set the upper and lower limits of the reference value of the higher pitch in the interval (the threshold is 0.3% in this paper), and the simplest frequency ratio b_i / a_i is calculated according to the Fig. 2 [30] below, where the variable a denotes the tone with lower frequency in the interval, and the variable b denotes the higher one. The specific calculation steps are showed in [30]:

According to the interval defined by the Twelve-tone equal temperament, taking 1st Large Group (hereinafter referred to A_1 as the sound group) as an example, the simplest frequency ratio of each interval is calculated according to the above process, as shown in Table 1 [30]:

As shown in Table 1, the simplest integer ratio of each interval frequency is calculated based on the root and crown frequency via an iterative algorithm, which clarifies the calculation process of the “Simplest frequency ratio” theory.

Remark 1. An iterative algorithm is designed to calculate the simplest integer ratio of interval frequencies, which accurately divides the consonance level of each interval, and improves the accuracy of the interval consonance calculation.

(2) *Consonance level classification*. Considering that the shortest frequency ratio of intervals cannot directly define the consonance of any two tones via the simplest frequency ratio of each interval in Table 1 [30], variable d_i is introduced as a parameter to measure the consonance degree of intervals. The expression of d_i is shown in Eq. (1) below. The order of magnitude of the variable d_i is used to evaluate the consonance level of each interval as 1–4, and the results are shown in Table 2 [30], satisfying the basic principle that the larger d_i is, the more consonant the interval becomes.

$$d_i = 1\Big/\left(a_i^2 + b_i^2\right) \tag{1}$$

It can be seen from Table 2 [30] that the degree of concord calculated in this paper conforms to the traditionally recognized description of the degree of concord, indicating the effectiveness of the iterative algorithm.

(3) *Consonance equation fitting*. Regarding the variable d_i that cannot represent the degree of consonance of any two tones, according to the above-mentioned consonance level, a function fitting method is used to quantify interval consonance into the degree of consonance. The frequency difference u is set as the independent variable, and $d_i^{1/4}$ is set as the dependent variable y. Based on the least square method, the expression of interval consonance y with respect to the frequency difference u is as follows:

$$y = f_1(u) = \begin{cases} -0.2852u + 0.8409, \quad u < 2.2\,\text{Hz} \\ 1.2457\times10^{-4}u^3 - 0.0039u^2 + 0.0595u + 0.1004, \quad 2.2\,\text{Hz} \le u < 11\,\text{Hz} \\ -0.0076u^3 + 0.3505u^2 - 5.2607u + 26.0117, \quad 11\,\text{Hz} \le u < 19.8\,\text{Hz} \\ -0.0096u^2 + 0.4424u - 4.6619, \quad 19.8\,\text{Hz} \le u < 26.4\,\text{Hz} \\ 0.0199u^2 - 1.1274u + 16.2128, \quad 26.4\,\text{Hz} \le u < 33\,\text{Hz} \\ 0.6687, \quad u \ge 33\,\text{Hz} \end{cases} \tag{2}$$

According to Eq. (2), the curve of the degree of consonance y with respect to the interval frequency difference u can be obtained by function fitting method as shown in Fig. 3 [30].

It can be seen from the Fig. 3 [30] below that the variation curve of the pure pitch consonance of a single sound group with the frequency difference is a broken line, and the pitch frequency difference close to

Perfect Unison and Perfect Octave corresponds to the highest consonance. Based on the least square method, the pure-tone interval consonance curve is fitted, and the consonance degree of any two tone in a single sound group is quantitatively described.

The consonance degree of each interval within a single sound group is established above. Due to the nonlinearity of pitch perception between different sound groups, Eq. (2) cannot directly describe the interval consonance of other sound groups. In this paper, the frequency difference normalization method is used to construct the interval consonance model of different sound groups, while the parameter adaptive method is applied to establish a universal interval consonance model considering the uncertainty of subjective differences.

The corresponding frequency difference is distinct in different tone groups in each sound group of the same interval, and as the frequency range of the sound group expands, the corresponding frequency difference will increase, resulting in the interval frequency difference outside the sound group not in the definition domain of Eq. (2). Based on the relationship between frequency difference in different sound groups, the root note is introduced to distinguish the sound group in which the interval is located, then the expression of interval on frequency difference and root note is obtained as follows:

$$u = g(x, a_1) = x/(a_1/33) = 33x/a_1 \tag{3}$$

where frequency difference x is $x = a_2 - a_1$, a_2 denotes crown note, a_1 denotes root note.

After the normalization of interval frequency difference, Eq. (2) can be used to calculate interval concord in different sound groups.

Considering the nonlinearity of auditory perception, the interval consonance increases with the increase of sound groups frequency, then logarithmic linearization coefficient $\eta(\bar{a})$ is introduced to obtain the expression of interval consonance y in different sound groups, so as to improve the accuracy of the consonance model,

$$y = f(u, \bar{a}) = f_1(u)\eta(\bar{a}) = f_1(u) \cdot \left[\alpha/\lg 50 \cdot \lg(\bar{a}/1000) + 1\right] \tag{4}$$

where $\bar{a} = (a_1 + a_2)/2$, α denotes consonance degree linear gain (the initial value is set as 0.2, which involves later calculation), satisfying $0 < \alpha < 1$, considering the audible frequency range and the intermediate frequency range, $\bar{a}$ satisfies the range $20 \le \bar{a} \le 20\,000$.

The perturbation of the logarithmic linearization coefficient $\eta(\bar{a})$ is caused by the uncertainty of the subjective differences. To solve this problem, based on the comprehensive database of professional/non-professional listeners, the parameter adaptive method is adopted to optimize the interval consonance model to quantify the subjective characteristics of consonance [23]. The specific steps are shown as Algorithm 2 [30], where N denotes the number of experimental samples, y_1 and y_2 denote the original consonance degrees of two sound groups, respectively, λ denotes consonance degree difference between two sound groups, $\bar{y}_1$ and $\bar{y}_2$ denote the consonance of two sound groups estimated by the listener, respectively, $\hat{\lambda}$ denotes the estimated consonance degree difference between two sound groups, k denotes the impact factor, which depends on the listener's understanding of the music and the level of expertise, satisfying that the sense of music is proportional to the value of k. Based on the above parameter adjustment method, the research experiment is conducted in this paper, and the linear gain $\alpha = 0.48553$ is determined. The specific calculation steps of the parameter adaptive are shown in [30].

Remark 2. The adaptive technique is used to quantify the uncertainty of subjective differences, and the parameters of the consonance model are adjusted to optimize the consonance calculation process.

The detailed experimental process is shown in Section 4.1.

In this section, a calculation method of interval frequency integer ratio based on iterative algorithm is proposed to quantitatively describe "The simplest frequency ratio" theory; The frequency difference between each sound group results in the small range of application for the consonance model of pure-tone interval within a single tone group, the model is then improved via frequency difference normalization method; The parameter of the consonance model of pure-tone interval is optimized by adaptive method, and the uncertainty of subjective difference is accurately described.

2.2. Complex-tone interval consonance degree calculation

The actual sound effect of music is determined by the complex sound composed of pure-tone and overtone, which is the multiple harmonics of the fundamental wave corresponding to pure-tone.

In the previous section, a pure-tone interval consonance model was constructed based on the frequency difference of pure-tone sound groups and the uncertainty of consonance. During the calculation of interval consonance, the missing harmonic factors will lead to the decrease of model accuracy [24]. In this paper, the consonance degree of multiple harmonic pairs is calculated, and the mixed density matrix is introduced to optimize the model.

Harmonic pair refers to the frequency combination of two harmonics corresponding to intervals in accordance with certain rules. The selection of harmonic pair should follow the steps below:

(1) List the fundamental and harmonic frequencies of the root note in the interval;

(2) Take the fundamental frequency n_1 of the crown sound, and find the frequency m_1 closest to n_1 in the fundamental and harmonic frequencies of the root note, then n_1 and m_1 form the first harmonic pair;

(3) Take the second harmonic frequency n_2 of the crown sound, and find the frequency m_2 closest to n_2 in the fundamental and harmonic frequency of the root sound. n_2 and m_2 form the second harmonic pair;

(4) Follow step 3 and continue until 4 harmonic pairs are found.

These 4 harmonic pairs are regarded as 4 intervals, and the consonance degree is calculated by Eq. (4), then the consonance degrees y_1, y_2, y_3 and y_4 of the 4 harmonic pairs are obtained, respectively.

For the overtone series of intervals, the intensity of high-order overtones is relatively low, which has little influence on the overall consonance degree, and its influence shows a decreasing trend with the increase of the order [25]. Therefore, the weight coefficients of 0.8, 0.1, 0.07 and 0.03 are, respectively introduced according to the order of harmonic pairs.

Considering the superposition property of pure-tone and overtone, the probability property of quantum state is used to further optimize the above model, so as to obtain a more universal interval consonance model. First, normalize the result of Eq. (4) and introduce the eigenstate probability amplitude $\sqrt{\mu}$, the expression is as follows:

$$\mu = \left(y - f_{\min}\right) / \left(f_{\max} - f_{\min}\right) \tag{5}$$

Convert the pure-tone interval consonance into a quantum state calculation model:

$$|\phi\rangle = \sqrt{\mu}\,|1\rangle + \sqrt{1-\mu}\,|0\rangle \tag{6}$$

where state $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents perfect consonance, and state $|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents imperfect consonance, that is, the quantum state $|\phi\rangle$ can be expressed as a vector form $\begin{pmatrix} \sqrt{\mu} \\ \sqrt{1-\mu} \end{pmatrix}$.

Considering the fourth harmonics, the harmonic consonance degree $|\phi_i\rangle$ corresponds to the coefficient μ_i ($i = 1,2,3,4$). Since the existence of overtones cannot be ignored, the chord consonance $|\phi\rangle$ is considered to be a mixed state formed by incoherent mixing of several pure states $|\phi_i\rangle$ in the system, where the probability of occurrence is P_i, that is, the weight coefficient corresponding to the i-th harmonic pair. In order to describe the mixed state, the density matrix ρ is introduced here. First, according to Eq. (6), the pure state density matrix can be expressed as [22]:

$$\rho_i = |\phi_i\rangle\langle\phi_i| = \begin{pmatrix} \mu_i & \sqrt{\mu_i(1-\mu_i)} \\ \sqrt{\mu_i(1-\mu_i)} & 1-\mu_i \end{pmatrix} \tag{7}$$

By expanding Eq. (7), it can be obtained as

$$\rho = \sum P_i\,|\phi_i\rangle\langle\phi_i| \tag{8}$$

where P_i is set as 0.8, 0.1, 0.07 and 0.03 respectively, according to Eq. (5). By substituting Eq. (6) into Eq. (7), the overall consonance degree expression can be obtained as follows,

$$\rho=\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix} \tag{9}$$

where

$$\rho_{11}=0.8\mu_1+0.1\mu_2+0.07\mu_3+0.03\mu_4 \tag{10}$$

$$\rho_{12}=\rho_{21}=0.8\sqrt{\mu_1(1-\mu_1)}+0.1\sqrt{\mu_2(1-\mu_2)}+0.07\sqrt{\mu_3(1-\mu_3)}+0.03\sqrt{\mu_4(1-\mu_4)} \tag{11}$$

$$\rho_{22}=1-(0.8\mu_1+0.1\mu_2+0.07\mu_3+0.03\mu_4) \tag{12}$$

THEOREM 1. *If the trace of the overall consonance degree Eq. (9) is always 1, the quantum system is still pure, that is, the system composed of fundamental wave and harmonic wave is closed and decoupled from external interference*.

According to Eq. (9), the density matrix of the system is

$$\rho=\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix} \tag{13}$$

Then, the matrix ρ^2 is denoted by

$$\rho^2=\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}=\begin{pmatrix}\rho_{11}^2+\rho_{12}\rho_{21} & \rho_{21}\rho_{11}+\rho_{22}\rho_{21}\\ \rho_{21}\rho_{11}+\rho_{22}\rho_{21} & \rho_{21}\rho_{12}+\rho_{22}^2\end{pmatrix} \tag{14}$$

According to Equations Eq. (10), Eq. (11) and Eq. (12), trace calculation of matrix ρ^2 can be obtained as

$$\begin{aligned}tr(\rho^2)&=\rho_{11}^2+\rho_{22}^2+2\rho_{12}\rho_{21}=(0.8\mu_1+0.1\mu_2+0.07\mu_3+0.3\mu_4)^2+\left[1-(0.8\mu_1+0.1\mu_2+0.07\mu_3+0.3\mu_4)\right]^2\\&+2\times\left[0.8\sqrt{\mu_1(1-\mu_1)}+0.1\sqrt{\mu_2(1-\mu_2)}+0.07\sqrt{\mu_3(1-\mu_3)}+0.03\sqrt{\mu_4(1-\mu_4)}\right]^2\end{aligned} \tag{15}$$

It can be obtained from Eq. (15) that only in rare cases $tr(\rho^2)=1$ is true, then for the system composed of pure-tone intervals and overtone harmonic pairs, the quantum state corresponding to the consonance degree is not a simple probability amplitude superposition, but coexists in a more complex form. Therefore, the density matrix is introduced to describe the degree of consonance in the analysis of composite consonance.

The influence of harmonic factors in the interval system is considered in this section, a kind of harmonic pair selection rules are designed, and a complex-tone interval consonance model is constructed by calculating the consonance of multiple harmonic pairs and introducing the mixed-state density matrix.

3. CHORD CONSONANCE DEGREE

In the previous section, an interval consonance model was established. The chord consonance degree is determined by the consonance of any two intervals. The coupling between intervals is difficult to describe, which increases the difficulty of constructing the chord consonance model. Based on the above quantum state model of interval consonance, this section takes triad, seventh and ninth chords as the research object [15], and uses quantum logic gate to regard chord as the quantum state that takes interval as the initial state and evolves in chord system, and then puts forward the calculation model of chord consonance.

3.1. Triad consonance degree

The three tones in a triad are combined in pairs to form three mutually coupled intervals, which together determine the consonance of the triad. A quantum logic gate and consonance compensation matrix are introduced in this section to establish the triad consonance model based on the interval consonance quantum state model, providing research ideas for the construction of the seventh and ninth chord consonance models.

Triads consist of four kinds of chords: major triad, minor triad, augmented triad and diminished triad, the composition structures of which are presented in Table 3 [30]. The three tones of the triad are denoted by x_1, x_2, x_3 in order from low to high frequency. Since the low frequency note when the chord is emitted affects people's auditory experience more [27], it will have a greater impact on the overall consonance degree, the two low frequency notes x_1 and x_2 in the triad are taken as the basic interval for the consonance degree calculation here. According to Eq. (6), the interval consonance quantum state composed of x_1 and x_2 can be expressed as:

$$|\phi\rangle = \sqrt{\mu_{12}}\,|1\rangle + \sqrt{1-\mu_{12}}\,|0\rangle \tag{16}$$

where μ_{12} denotes the consonance degree of x_1 and x_2, the interpretation of $|1\rangle$ and $|0\rangle$ refer to Eq.(6); The vector form of Eq. (16) can be expressed as $|\phi\rangle = \begin{pmatrix} \sqrt{\mu_{12}} \\ \sqrt{1-\mu_{12}} \end{pmatrix}$.

For quantum systems that do not interact with the environment, their evolution process is unitary, while the evolution process of any quantum state can be called quantum logic gate [28].

In this paper, the quantum state of triad system is considered as the evolvement state from the fundamental interval x_1x_2 quantum state through the operation of double quantum revolving-gate, the unitary transformation matrix of quantum revolving-gate is described as follows:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{17}$$

s_1 and s_2 respectively represent the consonance of x_1, x_3 and the consonance of x_2, x_3, then the vector form of triad consonance degree can be expressed as:

$$c_t = \begin{pmatrix} c_{t1} \\ c_{t2} \end{pmatrix} = P \begin{pmatrix} \sqrt{s_1} & -\sqrt{1-s_1} \\ \sqrt{1-s_1} & \sqrt{s_1} \end{pmatrix} \begin{pmatrix} \sqrt{s_2} & -\sqrt{1-s_2} \\ \sqrt{1-s_2} & \sqrt{s_2} \end{pmatrix} \begin{pmatrix} \sqrt{\mu_{12}} \\ \sqrt{1-\mu_{12}} \end{pmatrix} \tag{18}$$

where the orthogonal matrix P denotes the consonance compensation matrix. According to the quantum state expression of consonance degree (6), the consonance degree of triad can be obtained as c_{t1}^2.

Considering the interval composition of the four triads, the concordances of major triads, minor triads, augmented triads and diminished triads are calculated according to Table 3 [30] and Eq. (18), the results of which are shown in Table 3 [30] (four decimal places are taken), where the consonance compensation matrix P is expressed as follows:

$$P = P_1P_2 = \begin{pmatrix} \sqrt{0.4} & \sqrt{0.6} \\ -\sqrt{0.6} & \sqrt{0.4} \end{pmatrix} \begin{pmatrix} \sqrt{0.5} & \sqrt{0.5} \\ -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \tag{19}$$

According to Table 3 [30], the consonance degree of major triad and minor triad is relatively large, indicating that major triad and minor triad are more consonant than augmented triad and diminished triad. Therefore, major triad and minor triad are considered as consonant chord, while augmented triad and diminished triad belong to disconsonant chord.

Remark 3. Based on the quantum state model of interval consonance, the quantum logic gates and the consonance compensation matrix are introduced to calculate the chord consonance, thereby the integral construction of chord consonance quantum state model is realized.

In this section, quantum logic gates are introduced to describe the coupling characteristics between the intervals of triads, so as to establish the triad consonance degree model. The calculated results of this model are in line with the recognized consonance in music, and the validity of Eq. (18) is verified.

3.2. Seventh and ninth chord consonance degree

The pairings of notes within the seventh and ninth chords produce more intervals, and the coupling between them is more complicated than that of the triad. In this section, based on the calculation model of triad consonance degree and referring to the construction mode of this model, quantum logic gate is introduced to calculate the seventh chord and the ninth chord consonance degree.

The seventh chord is composed of four notes superimposed, and the four notes in the seventh chord are represented as x_1, x_2, x_3 and x_4 in order of frequency from low to high. The three notes with lower frequency in the seventh chord are formed into triads, as can be obtained from Eq. (17), the consonance vector form of this triad can be expressed as $c_t = \begin{pmatrix} c_{t1} \\ c_{t2} \end{pmatrix}$. Meanwhile, the consonance degrees of interval x_1x_4, x_2x_4 and x_3x_4 are calculated as s_1, s_2 and s_3 respectively. By expanding Eq. (17), the vector form of seventh chord consonance degree can be written as follows:

$$c_s = \begin{pmatrix} c_{s1} \\ c_{s2} \end{pmatrix} = P \begin{pmatrix} \sqrt{s_1} & -\sqrt{1-s_1} \\ \sqrt{1-s_1} & \sqrt{s_1} \end{pmatrix} \begin{pmatrix} \sqrt{s_2} & -\sqrt{1-s_2} \\ \sqrt{1-s_2} & \sqrt{s_2} \end{pmatrix} \begin{pmatrix} \sqrt{s_3} & -\sqrt{1-s_3} \\ \sqrt{1-s_3} & \sqrt{s_3} \end{pmatrix} \begin{pmatrix} c_{t1} \\ c_{t2} \end{pmatrix} \tag{20}$$

$$P = P_1P_2P_3 = \begin{pmatrix} \sqrt{0.4} & \sqrt{0.6} \\ -\sqrt{0.6} & \sqrt{0.4} \end{pmatrix} \begin{pmatrix} \sqrt{0.5} & \sqrt{0.5} \\ -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} \sqrt{0.3} & \sqrt{0.7} \\ -\sqrt{0.7} & \sqrt{0.3} \end{pmatrix} \tag{21}$$

where the orthogonal matrix P denotes the consonance degree compensation matrix, whose expression is shown in Eq. (21). In view of Eq. (6), the consonance degree of seventh chord is $c_{s1}{}^2$.

The ninth chord is composed of five notes superimposed, and the five notes in the ninth chord are represented as x_1, x_2, x_3, x_4 and x_5 in order of frequency from low to high. The four notes with lower frequency in the ninth chord are formed into seventh chord, based on the above seventh chord consonance degree model, and considering the consonance degrees of intervals x_1x_5, x_2x_5, x_3x_5 and x_4x_5 as s_1, s_2, s_3 and s_4 respectively, the consonance degree vector form of ninth chord can be expressed as:

$$c_n = \begin{pmatrix} c_{n1} \\ c_{n2} \end{pmatrix} = P \left[\prod_{k=1}^{4} \begin{pmatrix} \sqrt{s_k} & -\sqrt{1-s_k} \\ \sqrt{1-s_k} & \sqrt{s_k} \end{pmatrix} \right] c_s \tag{22}$$

$$P = P_1P_2P_3P_4 = \begin{pmatrix} \sqrt{0.4} & \sqrt{0.6} \\ -\sqrt{0.6} & \sqrt{0.4} \end{pmatrix} \begin{pmatrix} \sqrt{0.5} & \sqrt{0.5} \\ -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} \sqrt{0.3} & \sqrt{0.7} \\ -\sqrt{0.7} & \sqrt{0.3} \end{pmatrix} \begin{pmatrix} \sqrt{0.3} & \sqrt{0.7} \\ -\sqrt{0.7} & \sqrt{0.3} \end{pmatrix} \tag{23}$$

where the orthogonal matrix P denotes the consonance degree compensation matrix, whose expression is shown in Eq. (23). In view of Eq. (6), the consonance degree of ninth chord is $c_{n1}{}^2$.

In conclusion, from the perspective of quantum information, the commonness of these three chords is revealed as follows: the quantum state of interval consonance is regarded as the initial state of the unitary evolution, while the chords are considered as the different stages of the whole system evolution, different chords are distinguished by the number of quantum revolving gates, thus form the research methods of chord consonance degree calculation model. This section analyzes the coupling characteristics between intervals in chords, and constructs the consonance degree model of triad, seventh and ninth chords. In addition, the experiment of section 4.2 shows that the chord consonance degree is consistent with the actual situation, and this model has certain validity.

4. EXPERIMENTS AND ANALYSIS

The experiments are divided into two groups: interval consonance and chord consonance, and the expression of interval consonance is adjusted by quantifying the subjective perception of 10 professional listeners and 16 non-professional listeners, aiming at the uncertainty of linear gain parameters in interval consonance caused by the nonlinearity of pitch perception. Meanwhile, the validity of the chord consonance degree calculation model is verified by experimenters' perception feedback on chord consonance.

4.1. Seventh and ninth chord consonance degree

In this section, an adaptive parameter adjustment method is adopted to approach the real value of the linear gain parameter for the interval consonance of different tone groups. The difference between the subjective perceived consonance degree of the experimenter and the consonance degree calculated by the original model is considered as feedback to adjust the parameters of the consonance degree model.

In this experiment, the subjective perception of interval consonance degree is quantified to make it match with the order of magnitude of the original consonance degree model, so as to realize the adaptive adjustment of model parameters. In this experiment, the listener's subjective perception of the level of interval consonance is quantified and matched with the order of magnitude of the original consonance model in order to achieve adaptive model parameter adjustment. The adaptive adjustment was carried out in accordance with the level of the interval consonance parameters perceived by the listeners in Table 4 [30], and a suitable parameter α was discovered to ensure that the interval consonance degree fitting function can converge to the values corresponding to different consonance levels in Table 4 [30] so that the parameter α could be used in the chord consonance calculation.

The listeners are selected as samples for their subjective feelings of the four intervals of 1st Large Group A_1 and unaccented octave A_3. The listeners rate the intervals they've heard with consonance level, and their subjective perception is quantified into four levels through the rating. The corresponding consonance degree is shown in Table 4 [30].

In the 1st Large Group A_1 and unaccented octave A_3, tritone, major sixth, perfect fifth, and perfect octave are selected as the listening materials. According to Eq. (4), the original value of the interval consonance of each sound group can be obtained. In view of the parameter adaptive process shown in Fig. 4 [30], the experimental samples are calculated and the results are processed by the average value. The linear gain between the 1st Large Group and unaccented octave is, then the Eq. 4 is expressed as

$$y = f_1(u)\left[0.48553/\lg 50 \cdot \lg(\bar{a}/1000) + 1\right] \tag{24}$$

Remark 4. The weight of professional listeners is set as three times that of non-professional listeners, that is, the weight of professionals is $k_1 = 0.3$ and the weight of non-professional listeners is $k_2 = 0.1$.

4.2. Chord consonance experiment

In this section, the validity of the chord consonance model proposed above is verified by experiments. The experimental content includes three different types of chord consonance models, namely triad, seventh chord and ninth chord, and the experimental subjects' feedback is used to compare with the calculated model respectively.

Same as Section 4.1, the experiment quantifies the subjective perception of the listener as an experimental sample and divides the harmony level into 4 levels, the corresponding consonance degree is shown in Table 5 [30].

Based on the above Table 5 [30], the experimental samples are calculated on a weighted average, and the chord consonance measured in the experiment follows the decreasing trend of triads, sevenths, and ninths. (Due to space limitations, the listener's ratings of the degree of chord harmony and all test values of chord harmony are not listed here).

According to Eq. (18), Eq. (20) and Eq. (22), the calculated values of partial chord consonance are shown in Table 6 [30]. Meanwhile, this paper will compare the proposed method with the triad consonance calculation method based on Helmholtz theory. The roughness or rapid beating between adjacent partials is related to the dissonance between two simultaneously played complex tones. The triad consonance formula is obtained by adjusting the weighting factor based on the intensities of the two pure tones. This model only quantifies triads without taking into account the coupling between overtones and intervals, leaving out the seventh and ninth chords. Furthermore, while the quantified values reflect the consonance values of different chords, the values of the harmonic are too close to tell apart the degree of consonance of different chords [29].

It is worth noting that the experimental values of chord consonance are not in the same order of magnitude as the calculated values obtained from the model in this paper. Therefore, when comparing, we mainly focus on the order of consonance in the same type of chord. In Table 6 [30], the calculated and experimental values of consonance degree of major and minor triads are larger than those of augmented and diminished triads, which are consistent with the traditionally recognized major and minor triads as consonant chords; The consonance of major triads and minor triads cannot be calculated according to the Helmholtz theory, the difference of other triad data is slight, and the change of interval harmony cannot be distinguished, which is inconsistent with the listener's actual auditory perception. In the comparison of the consonance degree of augmented triad and diminished triad, the consonance degree calculated according to Helmholtz theory is not as close to the actual value as the consonance degree calculated in this paper, indicating that the calculation method described in this paper can reflect the real consonance degree better. For the seventh chord, the calculated values show that the consonance degrees of the major seventh chord and the minor seventh chord are much larger than those of the minor-major seventh chord and the augmented-major seventh chord, which shows the same trend as the experimental values; As for the ninth chord, the calculated values of the major ninth chord and the minor ninth chord are the same, and the experimental values are also relatively close, which also indicates the consistency of the calculated values and the experimental values. Auditory perception is an essential feature of music art and is the human acoustic mechanism. The calculation of chord harmony needs to conform to this auditory mechanism. Suppose it matches the experimental data of auditory perception. In that case, it means that the calculation of consonance is more accurate and reasonable in music practice. Experimental data on the subjective perception of the listener is given in [30]. Overall, the chord consonance degree calculation model proposed in this paper can sort the consonance of the same type of chords and the chord consonance degree trend closest to the listener perception data.

The experiments in this section also verify the effectiveness of this method.

5. CONCLUSION

To address the nonlinearity of pitch frequency perception, the coupling and subjective uncertainty between intervals, the quantum adaptive method is adopted to establish the calculation model of chord consonance degree, so as to improve the tension and sense of hierarchy of musical works and reduce the ambiguity of traditional music emotion calculation. In this paper, an iterative algorithm is proposed to design the processing flow of the simplest interval frequency integer ratio, and the adaptive method is used to optimize the parameters of interval consonance, while the calculation of pure-tone consonance is completed. Harmonic pair selection rule is designed and its consonance degree is obtained, based on which the consonance degree of complex-tone interval can be calculated accurately by density matrix, meanwhile the quantum logic gate is introduced to describe the coupling between intervals, and the chord consonance degree is obtained based on the chord consonance degree formed by the musical tone pairings in the chord. A comparison experiment is designed to verify the method proposed in this paper shows higher accuracy. Building a chord consonance database using the harmonics degree modeling method proposed in this paper in conjunction with the development of artificial intelligence to optimize music generation and retrieval will be of great theoretical and engineering importance and value for music research in the future.

REFERENCES

1. Z.C. LONG, T. SHEN, Y. P. ZHANG, L. LUO, *Pitch shift in exsomatized cochlea observed by laser interferometry*, Chinese Physics Letters, **36**, *2*, art. 024302, 2019, DOI: 10.1088/0256-307x/36/2/024302.
2. Y. YOO, H. INWOOK, C. SEUNGMOON, *Consonance of vibrotactile chords*, IEEE Transactions on Haptics, **7**, *1*, pp. 3–13, 2014, DOI: 10.1109/TOH.2013.57.
3. M. SCHMIDMEIER, *2:3:4 Harmony within the tritave*, Journal of Mathematics and Music, **13**, *3*, pp. 222–244, 2019, DOI: 10.1080/17459737.2019.1605626.
4. J. WILD, *The computation behind consonance and dissonance*, Interdisciplinary Science Reviews, **27**, *4*, pp. 299–302, 2002, DOI: 10.1179/030801802225005662.

5. P.M.C. HARRISON, M.T. PEARCE, *Simultaneous consonance in music perception and composition*, Psychological Review, **127**, *2*, pp. 216−244, 2020. DOI: 10.1037/rev0000169.
6. A. RAMATHILAGAM, P. PITCHIPOO, *Modeling and development of fuzzy logic-based intelligent decision support system*, Romanian Journal of Information Science and Technology, **25**, *1*, pp. 58−79, 2022.
7. H.V. MCINTOSH, *Group theory and its applications*, Academic Press, 1975.
8. G. DILLON, *Calculating the dissonance of a chord according to Helmholtz theory*, The European Physical Journal Plus, **128**, *8*, art. 90, 2013, DOI: 10.1140/epjp/i2013-13090-4.
9. N.D. COOK, *Calculation of the acoustical properties of triadic harmonies*, The Journal of the Acoustical Society of America, **142**, *6*, pp. 3748−3755, 2017, DOI: 10.1121/1.5018342.
10. L.L. TRULLA, N. DI STEFANO, A. GIULIANI, *Computational approach to musical consonance and dissonance*, Frontiers in Psychology, **9**, art. 381, 2018, DOI: 10.3389/fpsyg.2018.00381.
11. G.M. BIDELMAN, M.G. HEINZ, *Auditory-nerve responses predict pitch attributes related to musical consonance-dissonance for normal and impaired hearing*, The Journal of the Acoustical Society of America, **130**, *3*, pp. 1488−1502, 2011, DOI: 10.1121/1.3605559.
12. D.L. BOWLING, D. PURVES, K.Z. GILL, *Vocal similarity predicts the relative attraction of musical chords*, Proceedings of the National Academy of Sciences, **115**, *1*, pp. 216−221, 2018, DOI: 10.1073/pnas.1713206115.
13. A.J. MILNE, S. HOLLAND, *Empirically testing Tonnetz, voice-leading, and spectral models of perceived triadic distance,* Journal of Mathematics and Music, **10**, *1*, pp. 59−85, 2016, DOI: 10.1080/17459737.2016.1152517.
14. J. CHEN, L. WANG, E. CHARBON, *A quantum-implementable neural network model*, Quantum Information Processing, **16**, *10*, art. 245, 2017, DOI: 10.1007/s11128-017-1692-x.
15. R. BLUTNER, *Modelling tonal attraction: tonal hierarchies, interval cycles, and quantum probabilities,* Soft Computing, **21**, *6*, pp. 1401−1419, 2017, DOI: 10.1007/s00500-015-1801-7.
16. Q. ZHANG, Y. WEI, *Particle swarm optimization with independent adaptive parameter adjustment*, Journal of Frontiers of Computer Science & Technology, **14**, *4*, pp. 637−648, 2020.
17. V.J.S. ARIAS, P.E. LLUIS, *Some remarks on hypergestural homology of spaces and its relation to classical homology*, Journal of Mathematics and Music, **14**, *3*, pp. 245−265, 2020, DOI: 10.1080/17459737.2020.1722269.
18. H. TONY, *Quantum computing: an introduction*, Computing Control Engineering Journal, **10**, *3*, pp. 105−112, 1999.
19. W. ANUKOOL, J. LIM, Y. SONG, J. AHN, *Quantum computing systems: a brief overview*, Journal of the Korean Physical Society, **73**, *6*, pp. 841−845, 2018, DOI: 10.3938/jkps.73.841.
20. M. GERGER, A. GÜMÜŞÇÜ, *Diagnosis of Parkinson's disease using spiral test based on pattern recognition*, Romanian Journal of Information Science and Technology, **25**, *1*, pp. 100−113, 2022.
21. D.L. BOWLING, D. PURVES, *A biological rationale for musical consonance*, Proceedings of the National Academy of Sciences, **112**, *36*, pp. 11155−11160, 2015. DOI: 10.1073/pnas.1505768112.
22. G. GENUYS, *Pseudo-distances between chords of different cardinality on generalized voice-leading spaces*, Journal of Mathematics and Music, **13**, *3*, pp. 193−206, 2019, DOI: 10.1080/17459737.2019.1622809.
23. M.A.-K.M.A. SOLIMAN, R.M. ABOBAKR, *Linearly and quadratically separable classifiers using adaptive approach*, Journal of Computer Science and Technology, **26**, *5*, pp. 908−918, 2011, DOI: 10.1007/s11390-011-0188-x.
24. R. PARNCUTT, G. HAIR, *Consonance and dissonance in music theory and psychology: Disentangling dissonant dichotomies*, Journal of interdisciplinary music studies, **5**, *2*, art. 11050202, pp. 119−166, 2011.
25. E.S. SONG, Y.J. LIM, B. KIM, *A user-specific approach for comfortable application of advanced 3D CAD/CAM technique in dental environments using the harmonic series noise model*, Applied Sciences, **9**, *20*, art. 4307, 2019, DOI: 10.3390/app9204307.
26. N. JACOBY, N. TISHBY, D. TYMOCZKO, *An information theoretic approach to chord categorization and functional harmony*, Journal of New Music Research, **44**, *3*, pp. 219−244, 2015, DOI: 10.1080/09298215.2015.1036888.
27. R. PARNCUTT, *Pitch-class prevalence in plainchant, scale-degree consonance, and the origin of the rising leading tone*, Journal of New Music Research, **48**, *5*, pp. 434−448, 2019, DOI: 10.1080/09298215.2019.1642360.
28. A.A. GORBATSEVICH, N. M. SHUBIN, *Quantum logic gates*, Physics-Uspekhi, **61**, *11*, pp. 1100−1115, 2018.
29. G. DILLON, *Calculating the dissonance of a chord according to Helmholtz theory*, The European Physical Journal Plus, **128**, *8*, art. 90, 2013.
30. H. XIAO, C. FUYANG, R. TIAN, FENG. GAN, *Quantum adaptive approach to chord consonance modeling* (supplementary material), Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science, https://acad.ro/sectii2002/proceedings/doc2023-2/09-Han_Supplementary_material_QACM.pdf, accessed in June 2023.

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