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NEUROADAPTIVE TRACKING CONTROL OF UNCERTAIN NONLINEAR SYSTEMS WITH SPATIOTEMPORAL CONSTRAINTS

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Abstract. This study investigates the neuroadaptive tracking control problem for a class of strict-feedback nonlinear systems with spatiotemporal constraints. An adaptive neural network-based control system is developed to alleviate the effects of modeling uncertainties and external disturbances. In particular, the proposed method ensures that the system tracking error has a predefined performance boundary (spatial constraint). Moreover, using a novel time-scale transformation method, uncertain nonlinear systems can achieve a prescribed finite-time convergence to a time-varying scaling function in the pointing position (temporal constraint). Finally, the efficiency of the proposed method is verified with two simulation examples.

Key words: spatiotemporal constraints, neuroadaptive, uncertain nonlinear systems, time-varying scaling function.

1. INTRODUCTION

Many practical engineering applications urgently require quick and accurate target control [33, 34]. For example, a vehicle must be capable of quickly navigating potentially accident-prone situations to avoid casualties. Regarding the tracking problem, the designed missile is expected to quickly track and hit the target; regarding drone performance, each device must arrive at a fixed location and at a specific time (see References [1, 2]). The traditional asymptotically stable method can no longer meet the current needs. Therefore, finite-time and fixed-time control methods have been developed, and they have gained considerable popularity over the past three decades. References [3–14] and the studies cited therein present some representative results on finite-time and fixed-time control. References [3, 5, 9] focus on the finite-time and fixed-time control of nonlinear systems under various conditions (e.g., state constraints, dead zone, and input saturation). To avoid the explosion of complexity problem, a dynamic surface control strategy is introduced in Reference [4]. In References [8, 10, 11, 13, 14], the finite-time and fixed-time control were extended to multi-agent control. Note, however, that in the results obtained in the above-mentioned literature, the settling time is related to the initial state and other design parameters. If the system's initial state is far from equilibrium, the convergence time will be very long. Design parameters other than the settling time also increase the computational complexity of the control device. In 2016, Song *et al.* [15] developed a prescribed time control strategy that effectively solved the above problems. The basic design idea involves system transformation: that is, find a time-varying function that can grow to infinity at the expected time and use this function to transform the system into a new system.

More recently, many advanced works have proposed the idea of a prescribed time control algorithm and applied it to, for example, multi-agent systems, normal form system, and strict-feedback-like system [16–18, 35]. However, the above design method only reflects the dynamic change of the system state from the initial time to the desired time T, and the movement after time T cannot be known. This strategy is therefore only feasible for some practical problems, such as the behavior of a missile that hits a target and the case in which $t \in \infty$ is considered in [19,20]. Because most practical systems have unknown parameters and uncertain external disturbances, it is difficult and expensive to achieve zero steady-state error when the control process is stable. In practical applications, a control with sufficient steady-state accuracy is acceptable. Therefore, the method discussed in [16–20] is no longer applicable, so a practical prescribed time control is proposed; this strategy has been studied by only a few scholars. Zhao *et al.* [21] applied the practical prescribed time control to normal-form nonaffine systems. Tan *et al.* [22] applied it to Euler- Lagrange systems. Cao *et al.* [23] considered partial or full state constraints.

Owing to physical conditions, technology, and security, the system state is inevitably spatially constrained during operation. For example, the operation trajectory of the end joint of an industrial manipulator is generally limited to the first quadrant of the Cartesian coordinate system. If the spatial constraints are not met, the inertia matrix will appear singular, affecting industrial production. Similar problems may also arise in various defense and aerospace systems. Therefore, studying the spatial (state/output) constraints control problem of nonlinear systems is of practical significance. Nevertheless, developing relevant spatial constraint control algorithms is challenging. A nonlinear system control algorithm based on a restricted Lyapunov function is proposed in the literature [27] and ensures that the output state of the system is maintained within a predetermined space constraint. This scheme is then extended to the strict feedback system [28]. In this method, the state constraint problem needs to be transformed into the error constraint problem so that it is within the predetermined range of the state; this is a conservative approach. To solve this problem, the integral-constrained Lyapunov function method is adopted in the literature [29], which only needs to ensure that the initial value of the state is within the predetermined range (rather than within the range of its subset). Various other spatial constraint methods have also been developed at the same time. For example, Ilchmann [30] and Bechlioulis [31] et al. further analyzed the above results and proposed funnel control and preset performance algorithms, which have received considerable research attention.

To the best of our knowledge, the neuroadaptive control of strict-feedback nonlinear systems with spatiotemporal constraints, which motivates this research, has not yet been made available. This study makes the following primary contributions:

- The proposed method allows user-defined performance by ensuring that the distance between agents' state trajectories and their reference state trajectories is less than the given bounds (spatial constraints).
- It achieves finite-time convergence to the position of a time-varying leader at a user-defined time (temporal constraints).
- In this study, incomplete system modeling and unknown interference problems are fully considered, and system stability tracking is ensured through the radial basis neural network and scale time-varying function.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1. System description

Consider a class of strict feedback nonlinear system described as

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_1 \\ \dots \\ \dot{x}_j = x_{j+1} + f_j(\bar{x}_j) + d_j \\ \dots \\ \dot{x}_n = u + f_n(\bar{x}_n) + d_n \\ y = x_1, j = 2, \dots, n-1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1,...,x_i]^\top \in R^i (i = 1,...,n)$ are system state vectors, $f_j(\bar{x}_j) \in \Re, j = 2,...,n$ is an unknown smooth nonlinear function, and $d_i \in \Re$ indicate unknown disturbance. $u \in \Re$ and $y \in \Re$ are control input and output, respectively.

Remark 1. System (1) represents a large class of nonlinear single-input single-output (SISO) systems. Furthermore, the electromechanical and flexible crane systems can be regarded as forms of system (1).

The desired trajectory of the tracking signal is defined as x_d, which is continuous and differentiable. The main control objective of this study is to design a neuroadaptive control strategy under spatiotemporal constraints such that:

1. the systematic tracking error satisfies spatiotemporal constraints;
2. all the signals in the closed-loop system are bounded.

2.2. Preliminaries

LEMMA 1 [24]. *Radial basis function neural networks (RBFNN) can approximate unknown continuous nonlinear functions $F(Z) : \Re^q \to \Re$ with arbitrary precision on a compact set $\Omega \subset \Re^q$.*

$$F(Z) = W^T S(Z) + \delta(Z) \tag{2}$$

where $W = [W_1, W_2, ..., W_n] \in R^l$ is the optimal weight of RBFNN, l is the neural network (NN) node number, $\delta(Z) \in \Re$ is the approximation error, satisfies $\mid \delta(Z) \mid \leq \bar{\delta}$, and $\bar{\delta} > 0$ is a constant. $S(Z) \in R^l$ is a known and bounded basis function, chosen as the Gaussian function form.

$$S_i(Z) = \exp[\frac{-(Z-\varsigma_i)^T(Z-\varsigma_i)}{\kappa_i^2}] \tag{3}$$

where $\varsigma_i = [\varsigma_1, \varsigma_2, ..., \varsigma_n]^T$ denotes the center of the receptive field, and κ_i represents the width of the Gaussian function.

ASSUMPTION 1 [25]. *The desired trajectory x_d and its $i(i = 1,...,n)-$order derivatives are known, continuous, and bounded. In addition, the system states are available for control design.*

Remark 2. For the condition in which the system state is available under assumption 1, if the system state cannot be used in the control design, a state observer must be constructed; however, this situation is not considered in this study.

Definition 1. A time constant T is set in advance so that the system tracking error converges within this T, that is, the temporal constraint; an upper bound ζ of the convergence error is set in advance so that it converges to ζ within the finite time T, that is, the spatial constraints.

3. CONTROL SCHEME

To adjust the settling time of the system, we introduce a time-varying piecewise function as follows:

$$\beta(t) = \begin{cases} \chi(\frac{T-t}{T})^n + \zeta, & 0 \leq t < T \\ \zeta, & t \geq T \end{cases} \tag{4}$$

where T and ζ represent the values of the predefined settlement time and convergence accuracy, respectively.

Remark 3. The temporal constraint value T and the spatial constraint value ζ in the spatiotemporal constraints are given in $\beta(t)$.

Define the system tracking error e as

$$e = y - x_d = x_1 - x_d \tag{5}$$

In order to design the following controller, we transform the tracking error e as follows:

$$\xi_1 = \tan(\frac{\pi}{2}\frac{e}{\beta}) \tag{6}$$

$$\xi_i = x_i - \alpha_{i-1}, i = 2,3,...,n \tag{7}$$

where α_{i-1} is a virtual controller. In this section, the control design is presented via the backstepping method, which is composed of n steps.

Step 1. It follows from (1), (5), (6), and (7) that

$$\dot{\xi}_1 = (1+\tan^2(\frac{\pi}{2}\frac{e}{\beta}))\frac{\pi}{2\beta}(x_2 + f_1(x_1) + d_1 - \frac{\dot{\beta}}{\beta}e) \tag{8}$$

Note that $\Pi = 1+\tan^2(\frac{\pi}{2}\frac{e}{\beta}))\frac{\pi}{2\beta}$, and from (7), one has

$$\dot{\xi}_1 = \Pi(\xi_2 + \alpha_1 + f_1(x_1) + d_1 - \frac{\dot{\beta}}{\beta}e) \tag{9}$$

Letting $F_1 = f_1(x_1) + d_1 - \frac{\dot{\beta}}{\beta}e$, and using Lemma 1, a RBFNN is adopted to estimate F_1 as:

$$F_1 = W_1^T S_1 + \delta_1 \tag{10}$$

Substituting (10) into (9) yields

$$\dot{\xi}_1 = \Pi(\xi_2 + \alpha_1 + W_1^T S_1 + \delta_1) \tag{11}$$

then

$$\xi_1\dot{\xi}_1 = \Pi\xi_1(\xi_2 + \alpha_1 + W_1^T S_1 + \delta_1) \tag{12}$$

In view of Young's inequality

$$\begin{aligned}
\xi_1(W_1^T S_1 + \delta_1) &\leq \mid \xi_1 \mid (\parallel W_1 \parallel \parallel S_1 \parallel + \bar{\delta}_1) \\
&\leq \mid \xi_1 \mid (\max\{\parallel W_1 \parallel, \bar{\delta}_1\} \cdot (1 + \parallel S_1 \parallel)) \\
&\leq \xi_1^2 \max{}^2\{\parallel W_1 \parallel, \bar{\delta}_1\} \cdot (1 + \parallel S_1 \parallel)^2 + \frac{1}{4} \\
&\leq \xi_1^2 \Theta_1 \Phi_1 + \frac{1}{4}
\end{aligned} \tag{13}$$

where $\Theta_1 = \max^2\{\parallel W_1 \parallel, \bar{\delta}_1\}$ and $\Phi_1 = \cdot(1 + \parallel S_1 \parallel)^2$. Consider the following Lyapunov function candidate as:

$$V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2r_1}(\Theta_1 - \hat{\Theta}_1)^2 \tag{14}$$

where $r_1 > 0$ is the design parameter. Then, the time derivative of V_1 along (12) and (13) can be derived as:

$$\dot{V}_1 \leq \Pi(\xi_1\xi_2 + \xi_1\alpha_1 + \xi_1^2\Theta_1\Phi_1 + \frac{1}{4}) - \frac{1}{r_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \tag{15}$$

Then, (15) can be rewritten as:

$$\dot{V}_1 \leq \Pi(\xi_1\xi_2 + \xi_1\alpha_1 + \xi_1^2\hat{\Theta}_1\Phi_1) + \frac{1}{r_1}\tilde{\Theta}_1(r_1\Pi\xi_1^2\Phi_1 - \dot{\hat{\Theta}}_1) + \frac{1}{4} \tag{16}$$

The virtual controller α_1 and adaptive law $\dot{\hat{\Theta}}_1$ are designed as:

$$\alpha_1 = -\frac{1}{\Pi}k_1\xi_1 - \xi_1\hat{\Theta}_1\Phi_1 \tag{17}$$

$$\dot{\hat{\Theta}}_1 = r_1\Pi\xi_1^2\Phi_1 - \sigma_1\hat{\Theta}_1 \tag{18}$$

where the parameters $k_1 > 0$, $r_1 > 0$, and $\sigma_1 > 0$ are chosen freely by the designer. $\hat{\Theta}_1$ is the estimation of Θ_1.

Remark 4. The parameters in the adaptive control law are freely set by the designer. However, in order to obtain better tracking performance, the designer needs to adjust these parameters.

Substituting (17) and (18) into (16) gives

$$\dot{V}_1 \leq -k_1\xi_1^2 + \frac{\sigma_1}{r_1}\tilde{\Theta}_1\hat{\Theta}_1 + \Pi\xi_1\xi_2 + \frac{1}{4} \tag{19}$$

Note that

$$\begin{aligned} \frac{\sigma_1}{r_1}\tilde{\Theta}_1\hat{\Theta}_1 &= \frac{\sigma_1}{r_1}(-\tilde{\Theta}_1^2 + \tilde{\Theta}_1\Theta) \\ &\leq -\frac{\sigma_1}{2r_1}\tilde{\Theta}_1^2 + \frac{\sigma_1}{2r_1}\Theta_1^2 \end{aligned} \tag{20}$$

Then, (19) can be rewritten as:

$$\dot{V}_1 \leq -k_1\xi_1^2 - \frac{\sigma_1}{2r_1}\tilde{\Theta}_1^2 + \Pi\xi_1\xi_2 + \Delta_1 \tag{21}$$

where $\Delta_1 = \frac{\sigma_1}{2r_1}\Theta_1^2 + \frac{1}{4}$, and $\Pi\xi_1\xi_2$ will be handled in the next step.

Step 2. By calculating (1) and (7), one has

$$\dot{\xi}_2 = \xi_3 + \alpha_2 + f_2(x_2) + d_2 - \dot{\alpha}_1 \tag{22}$$

Letting $F_2 = f_2(x_2) + d_2 - \dot{\alpha}_1$, and using Lemma 1, a RBFNN is adopted to estimate F_2 as:

$$F_2 = W_2^T S_2 + \delta_2 \tag{23}$$

By using Young's inequality, one has

$$\begin{aligned} \xi_2(W_2^T S_2 + \delta_2) &\leq |\ \xi_2\ |\ (\|\ W_2\ \|\|\ S_2\ \| + \bar{\delta}_2) \\ &\leq |\ \xi_2\ |\ (\max\{\|\ W_2\ \|, \bar{\delta}_2\} \cdot (1 + \|\ S_2\ \|)) \\ &\leq \xi_2^2 \max{}^2\{\|\ W_2\ \|, \bar{\delta}_2\} \cdot (1 + \|\ S_2\ \|)^2 + \frac{1}{4} \\ &\leq \xi_2^2\Theta_2\Phi_2 + \frac{1}{4} \end{aligned} \tag{24}$$

Further, the Lyapunov function is constructed as follows:

$$V_2 = V_1 + \frac{1}{2}\xi_2^2 + \frac{1}{2r_2}(\Theta_2 - \hat{\Theta}_2)^2 \tag{25}$$

then, the time derivative of V_2 along (22) and (24) is

$$\begin{aligned} \dot{V}_2 \leq &-k_1\xi_1^2 - \frac{\sigma_1}{2r_1}\tilde{\Theta}_1^2 + \Delta_1 + (\xi_2\xi_3 + \xi_2\alpha_2 + \xi_2^2\hat{\Theta}_2\Phi_2 \\ &+ \Pi\xi_1\xi_2) + \frac{1}{r_2}\tilde{\Theta}_2(r_2\xi_2^2\Phi_2 - \dot{\hat{\Theta}}_2) + \frac{1}{4} \end{aligned} \tag{26}$$

Then, the virtual controller α_2 and the adaptive law $\dot{\hat{\Theta}}_2$ are designed as:

$$\alpha_2 = -k_2\xi_2 - \xi_2\hat{\Theta}_2\Phi_2 - \Pi\xi_1 \tag{27}$$

$$\dot{\hat{\Theta}}_2 = r_2\xi_2^2\Phi_2 - \sigma_2\hat{\Theta}_2 \tag{28}$$

where the parameters $k_2 > 0$, $r_2 > 0$, and $\sigma_2 > 0$ are chosen freely by the designer.

With the help of (27) and (28), we have

$$\dot{V}_2 \le -k_1\xi_1^2 - \frac{\sigma_1}{2r_1}\tilde{\Theta}_1^2 + \Delta_1 - k_2\xi_2^2 + \frac{\sigma_2}{r_2}\tilde{\Theta}_2\hat{\Theta}_2 + \xi_2\xi_3 + \frac{1}{4} \tag{29}$$

Note that

$$\frac{\sigma_2}{r_2}\tilde{\Theta}_2\hat{\Theta}_2 \le -\frac{\sigma_2}{2r_2}\tilde{\Theta}_2^2 + \frac{\sigma_2}{2r_2}\Theta_2^2 \tag{30}$$

Then, we obtain

$$\dot{V}_2 \le -k_1\xi_1^2 - k_2\xi_2^2 - \frac{\sigma_1}{2r_1}\tilde{\Theta}_1^2 - \frac{\sigma_2}{2r_2}\tilde{\Theta}_2^2 + \xi_2\xi_3 + \Delta_2 \tag{31}$$

where, $\Delta_2 = \frac{\sigma_1}{2r_1}\Theta_1^2 + \frac{\sigma_2}{2r_2}\Theta_2^2 + \frac{1}{2}$, and $\xi_2\xi_3$ will be dealt with in the next step.

Step i ($i = 3, ..., n-1$). Choosing the Lyapunov function candidate as:

$$V_i = V_{i-1} + \frac{1}{2}\xi_i^2 + \frac{1}{2r_i}(\Theta_i - \hat{\Theta}_i)^2 \tag{32}$$

Similar to step 2, we can get

$$\alpha_i = -k_i\xi_i - \xi_i\hat{\Theta}_i\Phi_i - \xi_{i-1} \tag{33}$$

$$\dot{\hat{\Theta}}_i = r_i\xi_i^2\Phi_i - \sigma_i\hat{\Theta}_i \tag{34}$$

where k_i, r_i, and σ_i are positive design constants.

Substituting the values of α_i and $\dot{\hat{\Theta}}_i$ into the derivative of (32) yields

$$\dot{V}_i \le -\sum_{m=1}^{i} k_m\xi_m^2 - \sum_{m=1}^{i-1}\frac{\sigma_m}{2r_m}\tilde{\Theta}_m^2 + \Delta_{i-1} + \xi_i\xi_{i+1} + \frac{\sigma_i}{r_i}\tilde{\Theta}_i\hat{\Theta}_i + \frac{1}{4} \tag{35}$$

Note that

$$\frac{\sigma_i}{r_i}\tilde{\Theta}_i\hat{\Theta}_i \le -\frac{\sigma_i}{2r_i}\tilde{\Theta}_i^2 + \frac{\sigma_i}{2r_i}\Theta_i^2 \tag{36}$$

then

$$\dot{V}_i \le -\sum_{m=1}^{i} k_m\xi_m^2 - \sum_{m=1}^{i}\frac{\sigma_m}{2r_m}\tilde{\Theta}_m^2 + \xi_i\xi_{i+1} + \Delta_i \tag{37}$$

where $\Delta_i = \sum_{m=1}^{i}\frac{\sigma_m}{2r_m}\Theta_m^2 + \frac{i}{4}$, and $\xi_i\xi_{i+1}$ will be dealt with in the next step.

Step n. According to (1) and (7), one has

$$\dot{\xi}_n = u + f_n(x_n) + d_n - \dot{\alpha}_{n-1} \tag{38}$$

then

$$\xi_n\dot{\xi}_n = \xi_n(u + f_n(x_n) + d_n - \dot{\alpha}_{n-1}) \tag{39}$$

Letting $F_n = f_n(x_n) + d_n - \dot{\alpha}_{n-1}$, and using Lemma 1, a RBFNN is adopted to estimate F_n as:

$$F_n = W_n^T S_n + \delta_n \tag{40}$$

According to Young's inequality, we have

$$
\begin{aligned}
\xi_n(W_n^T S_n+\delta_n) &\leq |\, \xi_n \,| \,(\| W_n \| \| S_n \| + \bar{\delta}_n) \\
&\leq |\, \xi_n \,| \,(\max\{\| W_n \|, \bar{\delta}_n\} \cdot (1+ \| S_n \|)) \\
&\leq \xi_n^2 \max{}^2\{\| W_n \|, \bar{\delta}_n\} \cdot (1+ \| S_n \|)^2 + \frac{1}{4} \\
&\leq \xi_n^2 \Theta_n \Phi_n + \frac{1}{4}
\end{aligned} \tag{41}
$$

Choose the Lyapunov function candidate as:

$$
V_n = V_{n-1} + \frac{1}{2}\xi_n^2 + \frac{1}{2r_n}(\Theta_n - \hat{\Theta}_n)^2 \tag{42}
$$

then

$$
\dot{V}_n = \dot{V}_{n-1} + \xi_n \dot{\xi}_n - \frac{1}{r_n}\tilde{\Theta}_n \dot{\hat{\Theta}}_n \tag{43}
$$

Substituting (39) and (41) into (43), it follows that

$$
\begin{aligned}
\dot{V}_n \leq & -\sum_{m=1}^{n-1} k_m \xi_m^2 - \sum_{m=1}^{n-1} \frac{\sigma_m}{2r_m}\tilde{\Theta}_m^2 + \xi_{n-1}\xi_n + \Delta_{n-1} \\
& + \xi_n^2 \hat{\Theta}_n \Phi_n + \xi_n u + \frac{1}{r_n}\tilde{\Theta}_n (r_n \xi_n^2 \Phi_n - \dot{\hat{\Theta}}_n) + \frac{1}{4}
\end{aligned} \tag{44}
$$

Finally, the actual control input u and adaptive law $\dot{\hat{\Theta}}_n$ are designed as:

$$
u = -k_n \xi_n - \xi_n \hat{\Theta}_n \Phi_n - \xi_{n-1} \tag{45}
$$

$$
\dot{\hat{\Theta}}_n = r_n \xi_n^2 \Phi_n - \sigma_n \hat{\Theta}_n \tag{46}
$$

where k_n, r_n, and σ_n are positive design constants.

Substituting the values of u and $\dot{\hat{\Theta}}_n$ into (44) yields

$$
\begin{aligned}
\dot{V}_n \leq & -\sum_{m=1}^{n} k_m \xi_m^2 - \sum_{m=1}^{n-1} \frac{\sigma_m}{2r_m}\tilde{\Theta}_m^2 + \Delta_{n-1} \\
& + \frac{\sigma_n}{r_n}\tilde{\Theta}_n \hat{\Theta}_n + \frac{1}{4}
\end{aligned} \tag{47}
$$

Note that

$$
\frac{\sigma_n}{r_n}\tilde{\Theta}_n \hat{\Theta}_n \leq -\frac{\sigma_n}{2r_n}\tilde{\Theta}_n^2 + \frac{\sigma_n}{2r_n}\Theta_n^2 \tag{48}
$$

then

$$
V_n \leq -\sum_{m=1}^{n} k_m \xi_m^2 - \sum_{m=1}^{n} \frac{\sigma_m}{2r_m}\tilde{\Theta}_m^2 + \Delta_n \tag{49}
$$

where $\Delta_n = \sum_{m=1}^{n} \frac{\sigma_m}{2r_m}\Theta_m^2 + \frac{n}{4}$. Letting $\Upsilon = \min_{1 \leq m \leq n}\{2k_m, \sigma_m\}$ and $\Delta = \Delta_n$. Then we have

$$
V_n \leq -\Upsilon V + \Delta \tag{50}
$$

Then, the main results of this paper are summarized as the following theorem.

THEOREM 1. *Consider the strict-feedback nonlinear system (1). Suppose that Assumption 1 holds, and if the adaptive controller (45) is applied, then the following objectives are achieved.*

1. *The boundedness of all signals is ensured.*
2. *The systematic tracking error satisfies spatiotemporal constraints.*

Proof.

1. We first prove that the boundedness of all signals is ensured. According to (50), we can get $\Omega_1 = \{V||V| \leq \frac{\Delta}{\Upsilon}\}$, which means that V remains in a compact set after a finite time T_0, and signals ξ_i and $\tilde{\Theta}$ are ultimately uniformly bounded. In addition, the integral on both sides of (50) can be obtained by $V(t) \leq V(0) + \frac{\Delta}{\Upsilon}$, then $V \in \ell_\infty$ for any bounded initial condition, which implies $\xi_i \in \ell_\infty$, $\tilde{\Theta}_i \in \ell_\infty$, and $\hat{\Theta}_i \in \ell_\infty$. From (17), it is seen that $\alpha_1 \in \ell_\infty$, which further implies that $\alpha_i \in \ell_\infty$. As $\xi_i \in \ell_\infty$ and $\hat{\Theta}_i \in \ell_\infty$, then $u \in \ell_\infty$ and $\dot{\hat{\Theta}}_i \in \ell_\infty$ $(i = 1, 2, \ldots, n)$.
2. According to $\xi_1 \in \ell_\infty$, we can get $-\beta(t) < e < \beta(t)$. Then, it can be seen from (4) that in the interval $0 \leq t \leq T$, $\beta(t)$ is a monotonically decreasing function with respect to t, and the tracking error e satisfies $\Omega = \{e \in R :| e |< \zeta\}$. In addition, the size of the interval Ω and the convergent settlement time T are set in advance.

4. SIMULATION VERIFICATION

In order to further verify the effectiveness of the designed algorithm, this paper selects two examples.

4.1. Mathematical example

Consider the following system:

$$\begin{cases} \dot{x}_1 = x_2 + \cos(x_1) + d_1 \\ \dot{x}_2 = u + \cos(x_1 x_2) + d_2 \\ y = x_1 \end{cases} \tag{51}$$

where $f_1(x_1) = \cos(x_1)$, $f_2(\bar{x}_2) = \cos(x_1 x_2)$. The initial conditions are $x_1 = 0.5$, $x_2 = -1$. The uncertain disturbances are chosen as $d_1 = -0.5\exp(-x_1^2)$ and $d_2 = 0.02\cos(x_2 t)$. The desired trajectory is $x_d = 0.5\sin(t)$. The number of RBFNN neurons is selected as 8. The design parameters are chosen as $r_1 = r_2 = 0.5$, $\sigma_1 = \sigma_2 = 1$, $k_1 = 5$, $k_2 = 10$, $T = 4$, and $\zeta = 0.08$.

The resulting response curve is shown in Figs. 1–5. Figures 1 and 2 show the system output x_1, the desired trajectory x_d, and the tracking error e_1. Figures 3–5 show control input u, system states x_1 and x_2, and adaptive parameters $\hat{\Theta}_1$ and $\hat{\Theta}_2$, respectively. It can be seen from Fig. 2 that the designed control scheme makes the tracking error of the system satisfy spatiotemporal constraints, where the temporal constraint $T = 4$ and the spatial constraints $\zeta = 0.08$. This paper further verifies the superiority of the proposed method by comparing it with reference [26]. As can be seen from Figs. 1–3, the tracking error of the proposed method is smaller and the convergence rate is faster.

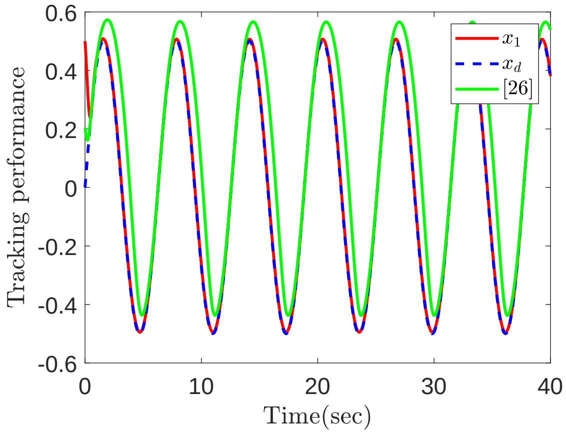

Fig. 1 – Curve of x_1 and x_d.

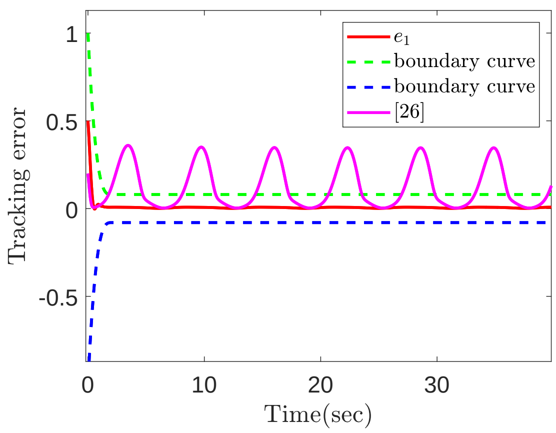

Fig. 2 – Curve of the tracking error e_1.

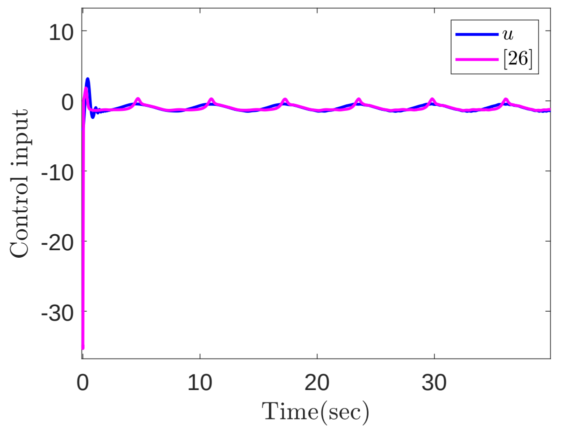

Fig. 3 – Curve of the controller u_1.

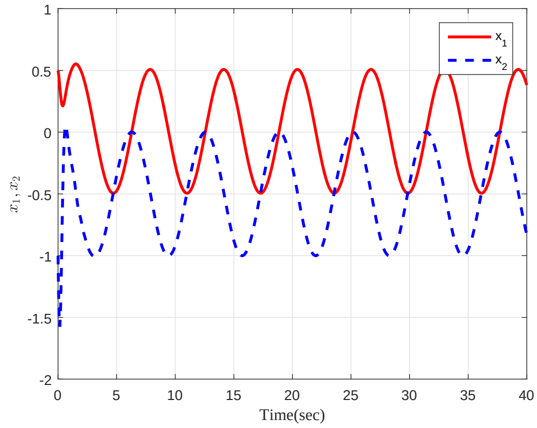

Fig. 4 – Curve of the states x_1 and x_2.

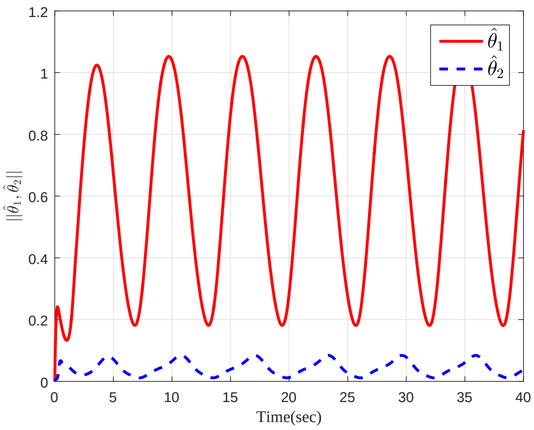

Fig. 5 – Curve of $\hat{\Theta}_1$ and $\hat{\Theta}_2$.

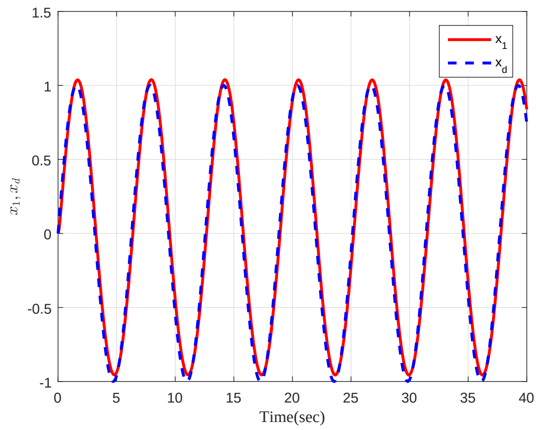

Fig. 6 – Curve of x_1 and x_d.

4.2. Physical example

We chose an actual electromechanical system, and its schematic diagram and system parameters are shown in References [32]. The system model expression can be expressed as:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(x_1,t) \\ \dot{x}_2 = \dfrac{1}{\frac{J}{K_\tau} + \frac{mL_0^2}{3K_\tau} + \frac{M_0L_0^2}{K_\tau} + \frac{2M_0L_0^2}{5K_\tau}}(x_3 - \frac{B_0}{K_\tau}x_2) \\ \qquad - \dfrac{\frac{mL_0G}{2K_\tau} + \frac{M_0L_0G}{K_\tau}}{\frac{J}{K_\tau} + \frac{mL_0^2}{3K_\tau} + \frac{M_0L_0^2}{K_\tau} + \frac{2M_0L_0^2}{5K_\tau}} \sin(x_1) + d_2(x_2,t) \\ \dot{x}_3 = \frac{1}{L}u - \frac{K_B}{L}x_2 - \frac{R}{L}x_3 + d_3(x_3,t) \\ y = x_1 \end{cases} \tag{52}$$

where R is the armature resistance, $d_i(x_i,t)$ is the unknown disturbance, L is the armature inductance, J is the rotor inertia, K_B is the back-emf coefficient, m is the link mass, G is the gravity coefficient, V_0 is the input control voltage, M_0 is the load mass, L_0 is the link length, R_0 is the radius of the load, B_0 is the coefficient of viscous friction at the joint, and K_τ is the coefficient that characterizes the electromechanical conversion of armature current to torque.

Choose the desired trajectory as $x_d = \sin(t)$. The number of RBFNN neurons is selected as 6. The design parameters are chosen as $r_1 = r_2 = r_3 = 1$, $\sigma_1 = \sigma_2 = \sigma_3 = 1$, $k_1 = 10$, $k_2 = 150$, $k_3 = 150$, $T = 5$, and $\zeta = 0.5$.

The resulting response curve is shown in Figs. 6–10. Figures 6 and 7 show system output x_1, desired trajectory x_d, and tracking error e_1. Figures 8–10 show control input u, system states x_1, x_2 and x_3, adaptive parameters $\hat{\Theta}_1$, $\hat{\Theta}_2$, and $\hat{\Theta}_3$, respectively. It can be seen from Fig. 7 that the designed control scheme makes the tracking error of the system satisfy spatiotemporal constraints, where the temporal constraint $T = 5$ and the spatial constraint $\zeta = 0.5$.

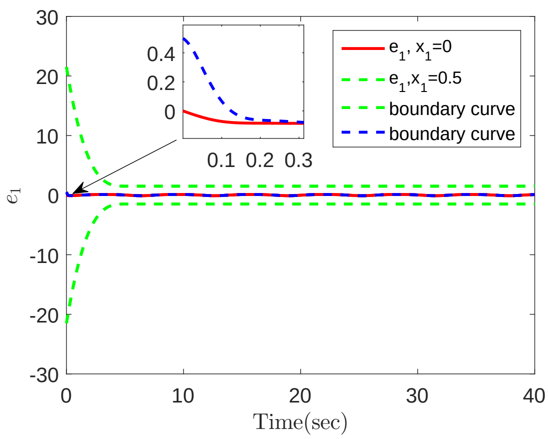

Fig. 7 – Tracking error e_1 curve under different initial conditions.

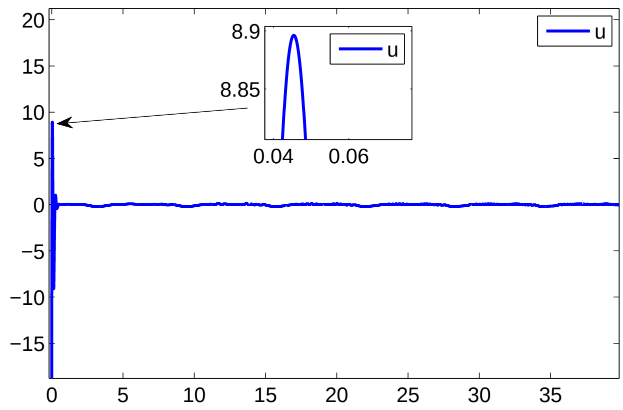

Fig. 8 – Curve of the controller u_1.

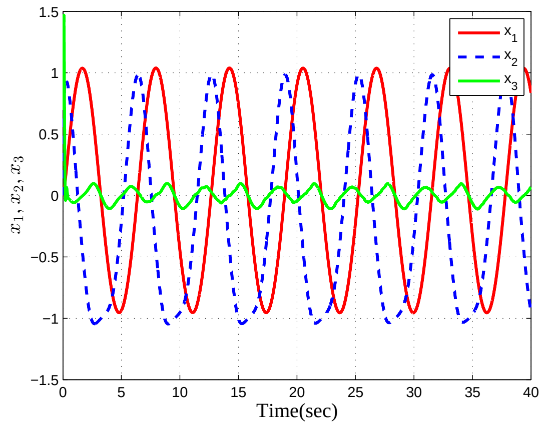

Fig. 9 – Curve of the states x_1, x_2 and x_3.

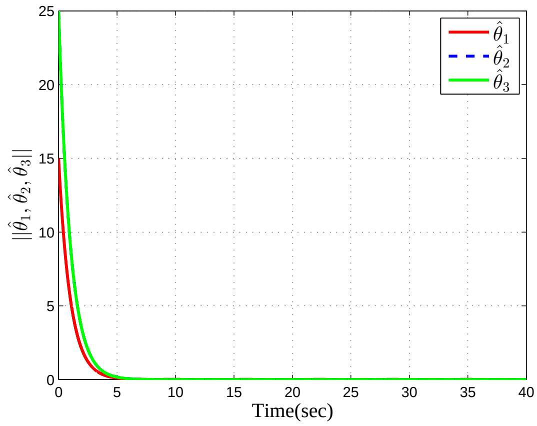

Fig. 10 – Curve of $\hat{\Theta}_1$, $\hat{\Theta}_2$ and $\hat{\Theta}_3$.

5. CONCLUSION

The proposed method combines an adaptive neural network with a scale-time-varying function so that the system tracking error satisfies the spatiotemporal constraints. The proposed scale-time-varying function provides the values of the temporal and spatial constraints in advance. These spatiotemporal constraints were therefore addressed regardless of the initial state of the system.

In the future, we aim to apply the spatiotemporal constraints to other systems, such as multi-agent systems, pure feedback systems, and multiple-input multiple-output systems. Another research avenue may involve applying the proposed method to practical engineering systems.

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